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# Photon mass generation during inflation: de Sitter invariant case

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We calculate the one-loop vacuum polarization tensor of scalar electrodynamics in a locally de Sitter space-time, endowed with a nearly minimally coupled, light scalar field. We show that the photon dynamics is well approximated by a (local) Proca Lagrangean. Since the photon mass can be much larger than the Hubble parameter, the photons may propagate slowly during inflation. Finally, we briefly discuss magnetic field generation on cosmological scales, and point out that, while the spectrum of the magnetic field is identical to that obtained from the massless scalar, $B_\ell \simeq B_0/\ell$, the amplitude $B_0$ may be significantly enhanced, implying that the seed field bound for the galactic dynamo can be easily met.

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## I. INTRODUCTION

In 1962 Julian Schwinger showed [1] that the photon of the 1+1 dimensional Quantum Electrodynamics (QED) acquires at one loop a mass $m_\gamma = e/\sqrt{\pi}$ as a consequence of the singular infrared behavior of the fermionic propagator. A little later, Anderson [2] showed that photons in conductive media may propagate as massive excitations. On the other hand, the gap generation in superconductors has as a consequence the Meissner effect, according to which static magnetic fields are exponentially suppressed within superconductors. Phenomenologically, the gap can be described by the scalar field condensate of the Landau-Ginzburg model, where the role of the scalar is taken by Cooper pairs. Abelian and non-Abelian gauge theories in thermal equilibrium have also been studied extensively [3]. It has been found that, while electric fields are Debye screened,

and cannot freely propagate, magnetic fields (at the perturbative level) do propagate as massless excitations, and screening is only dynamical. A realization that scalar condensates may be built by a fundamental scalar field, led to the establishment of the celebrated Higgs mechanism [4, 5, 6] (sometimes referred to as the Anderson-Englert-Brout-Higgs-Kibble mechanism), which currently represents the most popular explanation for the origin of the mass for the electroweak gauge bosons and fermions. This is of course not the whole story. A large part of the mass of mesons and baryons can be attributed to nonperturbative effects of a strongly coupled QCD. For an insightful historical overview of the development of our understanding of the concept of mass we refer to [7]. We paraphrase Okun: A tiny photon mass, albeit gauge non-invariant, does not destroy the renormalizability of QED [8, 9], and its presence would not spoil the beautiful agreement between QED and experiment. This motivates (so far unsuccessful) incessant searches for a nonvanishing photon mass [10, 11].

Quite recently, a novel mechanism for mass generation of gauge fields has been discovered [12, 13, 14, 15], which is operative in the presence of rapidly (superluminally) expanding background space-times. The concrete model within which the mechanism has been studied is scalar electrodynamics ($\Phi$QED), endowed with a massless, minimally coupled, scalar field evolving in a locally de Sitter space-time background. The photon mass is radiatively induced (at one loop) as a consequence of the canonical photon coupling to the amplified charged scalar fluctuations, which is in contrast to the Higgs mechanism, where the mass is induced by scalar condensate. We now compare this gravity induced mechanism with the case of thermal media. While magnetic fields get initially suppressed (with respect to the conformal vacuum), at asymptotically late times [15], just like in thermal equilibrium, they are *not* screened, and the final amplitude remains comparable to the conformal vacuum, $\vec{B} \sim \vec{B}_{\rm vac} \propto a^{-2}$, where $a$ denotes the scale factor. On the other hand, the photon mass in a rapidly expanding Universe implies exponentially enhanced (*anti-screened*) electric fields, $\vec{E} \propto a^{-3/2}$, which is in contrast to the Debye-screened electric fields of thermal media.

Here we reconsider the mass generation mechanism of Refs. [12, 13, 14, 15] in the context of massive scalar electrodynamics, whose Lagrangean density in a general metric field $g_{\mu\nu}$ reads,

$$\mathcal{L}_{\Phi\rm QED} = -\frac{1}{4}\sqrt{-g}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \sqrt{-g}g^{\mu\nu}(D_\mu\phi)^\dagger D_\nu\phi - \sqrt{-g}(m_\phi^2 + \xi R)\phi^*\phi\,, \tag{1}$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength, $D_\mu = \partial_\mu + ieA_\mu$ the covariant derivative, $g = \det[g_{\mu\nu}]$, and $R$ is the curvature scalar. Of course, through the Coleman-Weinberg mechanism [16], the effective $\Phi$QED acquires radiative corrections, which can be expressed through a renormalized

mass $m_\phi \to m_R$ and a renormalized quartic coupling, $\lambda_R$. Since we shall not study the consequences of radiatively induced symmetry breaking in the scalar sector, for our purposes it suffices to consider the physical effects of a nonvanishing scalar mass, $m_\phi^2 > 0$. We do however discuss briefly what happens in the case when $m_\phi^2 < 0$ (or, more precisely, when $m_\phi^2 + \xi R < 0$).

When recast in the locally de Sitter background,

$$g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}\,, \qquad \eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)\,, \tag{2}$$

where $a$ denotes the scale factor, which – when written in terms of conformal time $\eta$ – reads

$$a(\eta) = -\frac{1}{H\eta} \qquad (\eta < 0)\,, \tag{3}$$

$\mathcal{L}_{\Phi QED}$ in Eq. (1) reduces to,

$$\mathcal{L}_{\Phi QED} \longrightarrow -\frac{1}{4}a^{D-4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - a^{D-2}\eta^{\mu\nu}(\partial_\mu - ieA_\mu)\phi^*(\partial_\nu + ieA_\nu)\phi - a^D(m_\phi^2 + \xi R)\phi^*\phi\,, \tag{4}$$

where $H$ is the Hubble parameter in inflation, $\partial_\mu \equiv (\partial_\eta, \nabla)$, $D$ denotes the dimensionality of space-time, and we made use of $\sqrt{-g} \to a^D$. Recall that in D-dimensional de Sitter space-time, $R = D(D-1)H^2$. In this paper we shall assume a nearly minimally coupled, light scalar, for which $\xi \ll 1$ and $m_\phi^2 \ll H^2$. From Eq. (4) it immediately follows that, in $D = 4$, gauge fields are conformally invariant, while conformal invariance of scalar fields is granted for $\xi = 1/6$.

In section II we introduce the scalar two-point function $G = G(\bar{y})$, which (in $D$-dimensions and in the absence of electromagnetic fields) can be expressed in terms of the de Sitter invariant length, $\bar{y}$, as the following hypergeometric function,

$$G(\bar{y}) = \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{(4\pi)^{D/2}\Gamma(D/2)}H^{D-2}\,{}_2F_1\Big(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu, \frac{D}{2}; 1-\frac{\bar{y}}{4}\Big), \tag{5}$$

where $\nu$ is given in Eq. (20), $\bar{y} = \bar{y}(x;x')$ can be expressed in terms of the conformal coordinate interval, $\Delta\bar{x}^2(x;x') \equiv \eta^{\mu\nu}(x_\mu - x'_\mu)(x_\nu - x'_\nu) = -(\eta-\eta')^2 + \|\vec{x}-\vec{x}\,'\|^2$, and the geodesic distance $\ell = \ell(x;x')$,

$$\bar{y}(x;x') \equiv aa'H^2\Delta\bar{x}^2(x;x') = 4\sin^2\left(\frac{1}{2}H\ell(x;x')\right)\,. \tag{6}$$

where $a \equiv a(\eta), a' \equiv a(\eta')$.

It is instructive to expand the solution (5) around the minimally coupled case, such that in $D = 4$ we get,

$$G(\bar{y}) \overset{D\to 4}{\longrightarrow} \frac{H^2}{4\pi^2}\Big\{\frac{1}{\bar{y}} - \frac{1}{2}\ln(\bar{y}) + \frac{1}{2\mathtt{s}} - 1 + \ln(2) + O(\mathtt{s})\Big\}\,. \tag{7}$$

where the parameter $|\mathtt{s}| \ll 1$ is defined by,

$$\mathtt{s} \;\equiv\; \frac{D-1}{2} - \left[\left(\frac{D-1}{2}\right)^2 - \frac{m_\phi^2+\xi R}{H^2}\right]^{\frac{1}{2}} = \frac{m_\phi^2+\xi R}{(D-1)H^2} + O\Big(\big[(m_\phi^2+\xi R)/H^2\big]^2\Big)\,. \tag{8}$$

This is to be compared with the corresponding expression for the massless two-point function in $D=4$,

$$i\Delta(x;x')_{m=0} \;\;\stackrel{D\to 4}{\longrightarrow}\;\; \frac{H^2}{4\pi^2}\left\{\frac{\eta\eta'}{\Delta\bar{x}^2} - \frac{1}{2}\ln(H^2\Delta\bar{x}^2) - \frac{1}{4} + \ln(2)\right\}. \tag{9}$$

Apart from a constant term, the two solutions (7) and (9) differ by the term $[H^2/(8\pi^2)]\ln(aa')$ which is responsible for breakdown of the de Sitter invariance by the massless propagator [12, 15, 17, 18, 19, 20, 21], and it is a consequence of the growth of scalar fluctuations during inflation [22, 23]. Note that, in the limit when $\mathtt{s}\to 0$, the propagator (7) becomes formally singular, and hence ill defined. This is a consequence of the fact that it is *not* possible to construct a nontrivial de Sitter invariant propagator for a massless scalar [17, 21]. Note also that, since the non-Hadamard terms in the propagators (7) and (9) dominate in the infrared, they play a similar role in our mass generation mechanism as the fermionic states in the Schwinger mechanism, whose infrared behavior is sufficiently singular to lead to mass generation in $1+1$ dimensions.

In order to investigate how is the mass generation mechanism affected by the difference in the scalar propagators, in section III we outline our calculation of the one-loop, renormalized, vacuum polarization tensor of ΦQED to order $\mathtt{s}^0$. We then use standard techniques to arrive at the retarded tensor, which is then used in section IV to study the photon dynamics.

Our main result is presented in section IV, and can be summarized as follows. To order $\mathtt{s}^0$, the photon dynamics during inflation is governed by the Proca Lagrangean, which in D=4 reads,

$$\mathcal{L}_{\rm Proca} = -\frac{1}{4}\eta^{\mu\rho}\eta^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{1}{2}a^2 m_\gamma^2\eta^{\mu\nu}A_\mu A_\nu + O(\mathtt{s}^0) \tag{10}$$

with the photon mass-squared (accurate to order $\mathtt{s}^0$),

$$m_\gamma^2 \simeq \frac{\alpha H^2}{\pi\mathtt{s}} = \frac{3\alpha H^4}{\pi(m_\phi^2+\xi R)}\,, \tag{11}$$

where $\alpha = e^2/4\pi$ is the fine structure constant. The Proca Lagrangean is derived in a generalized Lorentz gauge,

$$\partial_\mu\big(a^2\eta^{\mu\nu}A_\nu\big) = 0\,. \tag{12}$$

This completely fixes the gauge, such that the photon contains two transverse and one longitudinal degree of freedom. In analogy to the Higgs mechanism, the longitudinal degree is supplied by the charged scalar fluctuations.

A consequence of the singular behavior of the two-point function (7) in the limit when $\mathtt{s} \to 0$ is an ill-defined photon mass (11). The correct treatment of the problem [12, 13] is then to use the massless propagator (9), in which case one recovers the (finite) result of Ref. [15], which can be interpreted as the (electric) photon mass, $m_\gamma^2 = (2\alpha/\pi)H^2 \ln(a)$. The time dependence is a consequence of the broken de Sitter invariance by the massless scalar propagator (9), which is $O(4)$ invariant. We expect that we would get a similar behaviour for a light nearly minimally coupled scalar, were we using a scalar propagator that is invariant only under the subgroup $O(4)$ of the de Sitter group $SO(4,1)$. The divergent behaviour of the photon mass found in Ref. [15] in the limit when $a \to \infty$ is related to the $m_\phi^2 + \xi\mathcal{R} \to 0$ behaviour of the photon mass (11) in the de Sitter invariant case.

Furthermore, it is interesting to compare the cases when $\mathtt{s}$ is small but positive, with the case when $\mathtt{s}$ is small but negative. In the case $1 \gg \mathtt{s} > 0$, the photon acquires a mass (11), and the photon amplitude oscillates and gets suppressed during inflation. On the other hand, when $-1 \ll \mathtt{s} < 0$, the mass-squared (11) becomes negative, indicating instability associated with a growth of the photon wave function. Consequently, when compared with the conformal vacuum, both electric and magnetic field can grow during inflation. This instability may imply primordial magnetic field generation of large amplitudes and on cosmological scales. The quantitative details of this mechanism will be addressed elsewhere. Another crucial difference between the mass generation in the case of a minimally coupled scalar and that of a near minimally coupled, light scalar is in the following. While in both cases the electric field is enhanced (anti-screened) with respect to the conformal vacuum, the magnetic field dynamics is strongly affected only in the latter case, in which the field is screened as $\vec{B} \propto a^{-\frac{5}{2}+\frac{1}{2}\sqrt{1-(m_\gamma/H)^2}}$.

Another noteworthy consequence of the photon mass generation is implied by considering the photon dispersion relation, which can be extracted from the Proca equation (obtained by the variation of Eq. (10)),

$$\eta^{\nu\rho}\partial_\rho(\partial_\nu A_\mu - \partial_\mu A_\nu) - a^2 m_\gamma^2 A_\mu = 0\,. \tag{13}$$

Assuming a dependence $e^{i\vec{k}\cdot\vec{x}}$ on the spatial coordinates and decomposing the photon field into transverse and longitudinal components, one obtains different equations of motion for these modes. However, in adiabatic limit, discussed in section IV A, which is valid for non-relativistic modes ($k_{\rm ph} \equiv \|\vec{k}\|/a \ll m_\gamma$) if $m_\gamma \gg H$ and for relativistic modes ($k_{\rm ph} \gg m_\gamma$) if $k_{\rm ph} \gg H$ (for longitudinal modes) or $k_{\rm ph} \gg (Hm_\gamma)^{1/2}$ (for transverse modes), one obtains the same dispersion relation

$$\omega \simeq \sqrt{\vec{k}^2 + a^2 m_\gamma^2} \tag{14}$$

for transverse and longitudinal modes. The propagation speed of massive photons is then given by the group velocity

$$\vec{v}_{\rm group} = \frac{d\omega_{\rm ph}}{d\vec{k}_{\rm ph}} \simeq \frac{\vec{k}_{\rm ph}}{\omega_{\rm ph}}\,, \qquad (15)$$

which, for $\|\vec{k}_{\rm ph}\| \ll m_\gamma$ can be $\ll 1$ (recall that in our units, the speed of light in *vacuo*, $c = 1$). Here we used the standard definitions for the physical frequency, $\omega_{\rm ph} \simeq \omega/a$, and the physical wave vector, $\vec{k}_{\rm ph} = \vec{k}/a$. Since, in the case when $0 < \mathtt{s} \ll 1$, $m_\gamma$ can be $\gg H$, a large class of both sub- and superhorizon photon modes may propagate with speeds, which are much smaller than the speed of light in *vacuo*. We have thus discovered that, provided inflation lasts a sufficiently long time, such that the de Sitter invariant solution for the scalar two-point function has had enough time to get established, light can propagate very slowly during inflation. Moreover, because the physical momentum scales as $\vec{k}_{\rm ph} \propto \vec{k}/a = \vec{k}\,{\rm e}^{-N}$, where $N = Ht$ denotes the number of e-foldings, $t$ the physical (cosmological) time, and $\vec{k}$ the conformal momentum (the physical momentum at time $t = 0$, or $\eta = -1/H$), the group velocity at asymptotically late times drops exponentially with time, $v_{\rm group} \propto {\rm e}^{-Ht}$ $(t \to \infty)$. We also note that, if the adiabatic conditions are not met, one can still calculate $\vec{v}_{\rm group} = d\omega/d\vec{k}$ by solving the equation for $\omega = \omega(\vec{k}, \eta)$ exactly. As an example, in section IV A we show that the relativistic longitudinal photons, for which $k_{\rm ph}, H \gg m_\gamma$, propagate superluminally.

The mass-induced slow-down of light differs significantly from the light slow-down reported not a long time ago [27] in ultra low temperature sodium gas, where the slow-down is induced by a steep frequency dependence of the index of refraction. Since the effect is of a resonant origin, it pertains only in a very narrow range of frequencies, which is in contrast to the mass-induced slow-down during inflation, which is effective for a broad range of frequencies. We also mention the work of Ref. [30], where an extreme version of the Fresnel effect is considered, in which superfluid vortices moving at a superluminal speed are used to generate an optical Aharonov-Bohm effect. Moreover, if they rotate faster than the speed of light in the medium, they can trap the slow light, and in this sense mimic the black holes of general relativity.

Finally, in section V we present an estimate of the cosmological magnetic fields produced as a consequence of a radiatively induced photon mass during inflation. Our conclusion is that the field strengths thus produced are sufficiently strong to satisfy the bounds on the seed magnetic field of the galactic dynamo mechanism. Appendices are reserved for technical details of the calculations.

## II. SCALAR PROPAGATOR

A complex scalar field of scalar electrodynamics (4) obeys (in the absence of electromagnetic fields) the following equation of motion,

$$\eta^{\mu\nu}\partial_\mu a^{D-2}\partial_\nu\phi - a^D(m_\phi^2+\xi R)\phi = 0. \tag{16}$$

Of course, the two-point Wightman functions, $\langle\alpha|\phi(x)\phi^\dagger(x')|\alpha\rangle$ and $\langle\alpha|\phi^\dagger(x')\phi(x)|\alpha\rangle$, satisfy the same differential equation. For a de Sitter invariant vacuum state $|\alpha\rangle$, where $\alpha$ represents the (real) parameter that classifies all globally de Sitter invariant vacua [21] (*cf.* also Refs. [31] and [32]), they can depend only on the geodesic distance between $x$ and $x'$, and may be written as a function of $z(x,x')\equiv 1-\bar{y}(x,x')/4$ only (*cf.* Eq. (6)). For such a function $\hat{G}(z)$ the differential equation can be recast as,

$$\frac{\eta^{\mu\nu}(\partial_\mu z)(\partial_\nu z)}{a^2}\frac{d^2}{dz^2}\hat{G} + \Big[\frac{\eta^{\mu\nu}\partial_\mu\partial_\nu z}{a^2} - \frac{(D-2)H(\partial_0 z)}{a}\Big]\frac{d}{dz}\hat{G} - (m_\phi^2+\xi R)\hat{G} = 0\,. \tag{17}$$

Upon calculating the derivatives of $z$, and contracting with $\eta^{\mu\nu}$, one obtains the hypergeometric differential equation,

$$z(1-z)\frac{d^2}{dz^2}\hat{G} + D\Big(\frac{1}{2}-z\Big)\frac{d}{dz}\hat{G} - (m_\phi^2+\xi R)\hat{G} = 0\,. \tag{18}$$

Taking account of the $z\Leftrightarrow 1-z$ symmetry of this equation, one can write the general solution for $G(\bar{y})\equiv\hat{G}(z)$ in terms of the hypergeometric functions [24, 25, 26],

$$G(\bar{y}) = c\;{}_2F_1\Big(\frac{D-1}{2}+\nu,\frac{D-1}{2}-\nu,\frac{D}{2};1-\frac{\bar{y}}{4}\Big) + c'\;{}_2F_1\Big(\frac{D-1}{2}+\nu,\frac{D-1}{2}-\nu,\frac{D}{2};\frac{\bar{y}}{4}\Big)\,, \tag{19}$$

where (so far) $c$ and $c'$ are arbitrary constants (which are dependent on $\alpha$), and

$$\nu = \bigg[\Big(\frac{D-1}{2}\Big)^2 - \frac{m_\phi^2+\xi R}{H^2}\bigg]^{\frac{1}{2}}\,. \tag{20}$$

The hypergeometric function becomes singular [21], when the last argument approaches 1 or $-1$, and has a branch cut from 1 ($-1$) to $+\infty$ ($-\infty$). In order to completely specify the Wightman functions, we demand that there be a singularity only if $x$ and $x'$ are light-like related. Furthermore, at short distances this singularity should have the Hadamard form of the Minkowski space two-point functions, since (at short distances) the scalar field can be only weakly affected by the expansion of space-time. With these assumptions one finds $c'=0$, such that (19) reduces to

$$G(\bar{y}) = \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})}H^{D-2}\;{}_2F_1\Big(\frac{D-1}{2}+\nu,\frac{D-1}{2}-\nu,\frac{D}{2};1-\frac{\bar{y}}{4}\Big)\,. \tag{21}$$

where the precise value of $c$ is dictated by the canonical commutation relation of $\phi$ and its canonical momentum [31]. This choice corresponds to the unique vacuum, $|\alpha = -\infty\rangle$. (The $\alpha = -\infty$ vacuum is in literature known under various names: Chernikov-Tagirov vacuum, Bunch-Davies vacuum, Euclidean vacuum, thermal vacuum.) The vacua with $\alpha \neq -\infty$ contain an additional singularity at the antipodal point, $\bar{y} = 4$ (which corresponds to $\eta' = -\eta$ and $\vec{x}\,' = \vec{x}$), which can be also interpreted as a constant ($\alpha$-dependent) nonvanishing particle number at all momenta, such that $\alpha \neq -\infty$ vacua are associated to a divergent stress-energy tensor. While this divergence should be regularized, it can be done so only for a particular choice of $\alpha$. In our point of view, the requirement that, at short distances, one should recover the Hadamard form for the singularity of two-point functions (which, by the way, has been tested in particle accelerators), singles out the $\alpha = -\infty$ vacuum in a natural way. Indeed, the Hadamard form can be violated only at the price of giving up locality in position space. The nonlocality is namely necessary if the Hubble scale physics is to influence the physics at much shorter scales, responsible for the Hadamard singularity.

Analogous to different pole prescriptions in the momentum space integrals for the various forms of the scalar propagator [44], one can write the (anti-)Feynman and Wightman propagators as the same function G of the appropriately modified de Sitter length functions,

$$i\Delta_{bb'}(x,x') = G(y_{bb'}) \qquad (b, b' = +, -)\,, \tag{22}$$

where

$$i\Delta(x,x') \equiv i\Delta_{++}(x,x') \equiv \langle 0|T[\phi(x)\phi^\dagger(x')]|0\rangle \qquad (\texttt{Feynman}) \tag{23}$$

$$i\Delta_{+-}(x,x') \equiv \langle 0|\phi^\dagger(x')\phi(x)|0\rangle \qquad (\texttt{Wightman}) \tag{24}$$

$$i\Delta_{-+}(x,x') \equiv \langle 0|\phi(x)\phi^\dagger(x')|0\rangle \qquad (\texttt{Wightman}) \tag{25}$$

$$i\Delta_{--}(x,x') \equiv \langle 0|\bar{T}[\phi(x)\phi^\dagger(x')]|0\rangle \qquad (\texttt{anti-Feynman})\,. \tag{26}$$

Here $T$ ($\bar{T}$) denotes time (anti-time) ordering and

$$y_{bb'} = \frac{\Delta x^2_{bb'}}{\eta\eta'}\,, \tag{27}$$

with

$$\Delta x^2 \equiv \Delta x^2_{++} = -\left(|\eta - \eta'| - i\delta\right)^2 + \|\vec{x} - \vec{x}'\|^2, \tag{28}$$

$$\Delta x^2_{+-} = -\left(\eta - \eta' + i\delta\right)^2 + \|\vec{x} - \vec{x}'\|^2, \tag{29}$$

$$\Delta x^2_{-+} = -\left(\eta - \eta' - i\delta\right)^2 + \|\vec{x} - \vec{x}'\|^2, \tag{30}$$

$$\Delta x^2_{--} = -\left(|\eta - \eta'| + i\delta\right)^2 + \|\vec{x} - \vec{x}'\|^2. \tag{31}$$

From now on we shall use the notation,

$$y \equiv y_{++} = \frac{\Delta x^2}{\eta\eta'} \,. \tag{32}$$

In order to make progress toward calculating the vacuum polarization tensor in dimensional regularization, we shall expand the scalar propagator (21)–(26) around the massless minimally coupled case and in the vicinity of $D = 4$. To this purpose, it is useful to define the parameter $\mathtt{s}$ as,

$$\mathtt{s} \equiv \frac{D-1}{2} - \nu, \tag{33}$$

where $\nu$ is defined in (20), and the parameter

$$\varepsilon \equiv 4 - D \,. \tag{34}$$

For a light and/or nearly minimally coupled scalar, for which $m_\phi \ll H$ and $|\xi| \ll 1$, we can expand $\nu$ in powers of $(m_\phi^2 + \xi R)/H^2$, to obtain the following approximation for $\mathtt{s}$,

$$\mathtt{s} = \frac{1}{D-1}\frac{m_\phi^2 + \xi R}{H^2} + O\left(\Big(\frac{m_\phi^2+\xi R}{H^2}\Big)^2\right) \,. \tag{35}$$

In Appendix A we show (see Eq. (A8)) that, when expanded in $\mathtt{s}$, $y$ and $\varepsilon$, the Feynman propagator (22–23) reduces to,

$$i\Delta(x,x') = \beta f(y) + \mathtt{s}\frac{H^{2-\varepsilon}}{(4\pi)^{2-\frac{\varepsilon}{2}}}\big[g(y) + h(y)\big] + O(\varepsilon, \mathtt{s}^2) \,, \tag{36}$$

where

$$\beta = \Big(\frac{H}{2\pi}\Big)^2\Big(\frac{\pi}{H^2}\Big)^{\frac{\varepsilon}{2}}\Gamma\Big(2-\frac{\varepsilon}{2}\Big), \tag{37}$$

$$f(y) = \frac{1}{1-\frac{\varepsilon}{2}}\frac{1}{y^{1-\frac{\varepsilon}{2}}} - \Big(1-\frac{\varepsilon}{4}\Big)\frac{y^{\frac{\varepsilon}{2}}}{\varepsilon} + \frac{2^\varepsilon\Gamma(3-\varepsilon)}{4\Gamma^2(2-\frac{\varepsilon}{2})}\Big[\frac{1}{\mathtt{s}} + \pi\cot\Big(\frac{\pi\varepsilon}{2}\Big) - \gamma_{\rm E} - \psi(3-\varepsilon)\Big], \tag{38}$$

$$g(y) = \frac{\Gamma(3-\varepsilon)}{\Gamma(2-\frac{\varepsilon}{2})}\frac{C(\mathtt{s},\varepsilon)}{\mathtt{s}} - \Big(\frac{y}{4}\Big)^{\frac{\varepsilon}{2}}\frac{\Gamma(3-\frac{\varepsilon}{2})}{\frac{\varepsilon}{2}}\Big[\pi\cot\Big(\frac{\pi\varepsilon}{2}\Big) - \psi\Big(3-\frac{\varepsilon}{2}\Big) + \psi\Big(\frac{\varepsilon}{2}\Big)\Big], \tag{39}$$

$$h(y) = \sum_{n=1}^{\infty}\frac{2-n(n+2)\ln(y/4)}{n^2}\Big(\frac{y}{4}\Big)^n \,. \tag{40}$$

Here $\Gamma$ denotes the Euler-Gamma function, $\gamma_{\rm E} \approx 0.577$ the Euler-Mascheroni constant, $\psi$ is defined by $\psi(z) \equiv \Gamma'(z)/\Gamma(z)$ and $C(\mathtt{s},\varepsilon)$ is an order $\mathtt{s}$ term that is independent on the coordinates. We will later find that it does not contribute to the vacuum polarization at order $\mathtt{s}^0$. The $O(\varepsilon)$ term in the infinite sum is not important, because it will not be needed for the dimensional regularization and renormalization of the vacuum polarization tensor outlined in section III and Appendix C.

For calculating the one-loop vacuum polarization tensor in a locally de Sitter space-time, we will also need the coincident limit $x' \to x$ (equivalently $y \to 0$) of the propagator (36). To obtain this limit in dimensional regularization, one must assume that $\varepsilon$ is large enough that a vanishing quantity like $y$ is raised to non-negative powers only. Thus we get

$$\lim_{x'\to x} i\Delta(x,x') = \beta \frac{2^{\varepsilon}\Gamma(3-\varepsilon)}{4\Gamma^2(2-\frac{\varepsilon}{2})}\Big[\frac{1}{\mathtt{s}} + \pi\cot\Big(\frac{\pi\varepsilon}{2}\Big) - \gamma_{\rm E} - \psi(3-\varepsilon)\Big] + O(\mathtt{s})\,. \tag{41}$$

This is in contrast with the massless propagator (9), which grows logarithmically with the scale factor during inflation, $i\Delta(x,x)_{m\to 0} \propto \ln(a)$.

## III. ONE-LOOP VACUUM POLARIZATION IN A LOCALLY DE SITTER SPACE

We are now ready to calculate the vacuum polarization tensor of scalar electrodynamics ($\Phi$QED) in curved space-time backgrounds (1), (4) in the one-loop approximation, based on which one can study the dynamics of photons during inflation (*cf.* Ref. [12]). The relevant diagrams contributing to the one-loop polarization tensor are shown in figure 1. The counterterm in figure 1.(3) is added in order to cancel the divergence, appearing in dimensional regularization in the limit when $D \to 4$. Using the position-space Feynman rules of Appendix B, one finds the following expressions for the one-loop graphs shown in figure 1

$$i[{}^{\mu}\Pi^{\nu}]^{(1)}(x,x') = -\,2ie^2\sqrt{-g(x)}g^{\mu\nu}(x)i\Delta(x,x)\delta^D(x-x'), \tag{42}$$

$$= -\,2ie^2 a^{D-2}\eta^{\mu\nu}i\Delta(x,x)\delta^D(x-x') \tag{43}$$

$$i[{}^{\mu}\Pi^{\nu}]^{(2)}(x,x') = 2e^2\sqrt{-g(x)}g^{\mu\rho}(x)\sqrt{-g(x')}g^{\nu\sigma}(x')$$

$$\times\big[(\partial_\rho i\Delta(x,x'))\partial'_\sigma i\Delta(x,x') - i\Delta(x,x')\partial_\rho\partial'_\sigma i\Delta(x,x')\big] \tag{44}$$

$$= 2e^2 a^{D-2}a'^{D-2}\eta^{\mu\rho}\eta^{\nu\sigma}\big[(\partial_\rho i\Delta(x,x'))(\partial'_\sigma i\Delta(x,x')) - i\Delta(x,x')\partial_\rho\partial'_\sigma i\Delta(x,x')\big], \tag{45}$$

where $i\Delta(x,x') = i\Delta(x',x)$ is used and

$$i[{}^{\mu}\Pi^{\nu}]^{(3)}(x,x') = -\,i\delta Z\partial_\rho\big(\sqrt{-g(x)}[g^{\mu\nu}(x)g^{\rho\sigma}(x) - g^{\mu\sigma}(x)g^{\nu\rho}(x)]\partial'_\sigma\delta^D(x-x')\big), \tag{46}$$

$$= -\,i\delta Z[{}^{\mu}P^{\nu}]a^{D-4}\delta^D(x-x'). \tag{47}$$

Here $a$ and $a'$ are defined by $a \equiv a(\eta)$, $a' \equiv a(\eta')$, and $[{}^{\mu}P^{\nu}]$ is the transverse projector defined in Eq. (49) below. One can use Eq. (41) for $i\Delta(x,x)$ in (43) and (36) for $i\Delta(x,x')$ in (45). Then the

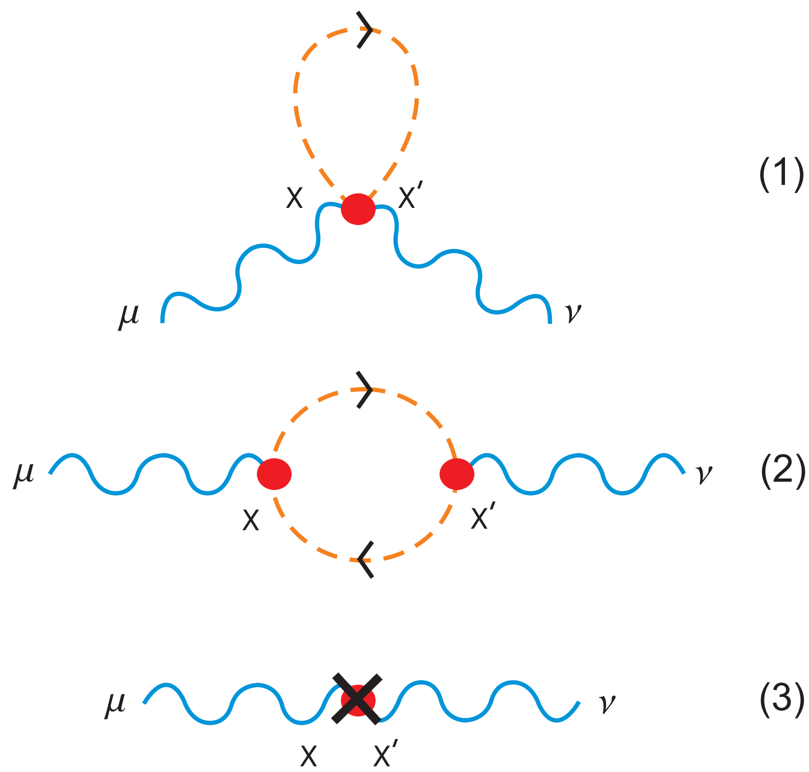


FIG. 1: The one-loop Feynman graphs contributing to the vacuum polarization tensor in scalar QED.

sum of the graphs Fig. 1.(1) and Fig. 1.(2), $i[^\mu\Pi^\nu_{1+2}]$ can be derived, one obtains (see Appendix C)

$$\begin{aligned} i[^\mu\Pi^\nu_{1+2}](x,x') &= \frac{\alpha}{2\pi^3}\Bigg\{\frac{\pi^\varepsilon\Gamma^2(1-\frac{\varepsilon}{2})}{2(3-\varepsilon)}\big[{}^\mu P^\nu\big]\frac{1}{\Delta x^{4-2\varepsilon}} - \big[{}^\mu P^\nu\big]\Bigg[\frac{\frac{1}{2}\ln(\frac{y}{4})-\frac{1}{2\mathtt{s}}+2}{\eta\eta'\Delta x^2} + \frac{\ln(\frac{y}{4})}{2(y-4)\eta^2\eta'^2}\Bigg] \\ &\quad + \big[{}^\mu\bar{P}^\nu\big]\frac{1}{\eta^2\eta'^2}\Bigg[\frac{1}{8}\ln^2\Big(\frac{y}{4}\Big) + \ln\Big(\frac{y}{4}\Big)\Big(1-\frac{1}{4\mathtt{s}}+\frac{y-2}{2(y-4)}\Big) - \frac{1}{4}Li_2\Big(1-\frac{y}{4}\Big)\Bigg] \\ &\quad + O(\mathtt{s})\Bigg\}, \end{aligned} \tag{48}$$

where

$$\begin{aligned} \big[{}^\mu P^\nu\big] &= \eta^{\mu\rho}\eta^{\nu\sigma}(\eta_{\rho\sigma}\partial'\cdot\partial - \partial'_\rho\partial_\sigma) \\ \big[{}^\mu\bar{P}^\nu\big] &= \eta^{\mu i}\eta^{\nu j}(\eta_{ij}\nabla'\cdot\nabla - \partial'_i\partial_j) \end{aligned} \tag{49}$$

are the manifestly transverse projector operators, $\partial_\mu[{}^\mu P^\nu] = 0 = \partial'_\nu[{}^\mu P^\nu]$, $\partial_\mu[{}^\mu \bar{P}^\nu] = 0 = \partial'_\nu[{}^\mu \bar{P}^\nu]$, and $\partial'\cdot\partial \equiv \eta^{\mu\nu}\partial'_\mu\partial_\nu$, $\nabla'\cdot\nabla \equiv \partial'_i\partial_i$ (greek indices $\alpha,\beta,..$ run from 0 to 3, while latin indices $i,j,..$ run from 1 to 3). In Eq. (48) D was taken to 4 in all terms, except in the first, because only this term requires regularization. $\Delta x^2$ and $y$ are given by (28) and (32), and $Li_2$ is the dilogarithm function defined by $Li_2(z) \equiv -\int_0^z \frac{\ln(1-t)}{t}dt$.

The first term in Eq. (48) is exactly what one would find for Fig.1.(1) and Fig. 1.(2) for a massless scalar field in flat space [12]. Its divergence for $\varepsilon \to 0$ can be seen from

$$\frac{1}{\Delta x^{4-2\varepsilon}} = -\frac{1}{2\varepsilon(1-\varepsilon)}\partial^2 \frac{1}{\Delta x^{2-2\varepsilon}}. \tag{50}$$

Combining this with

$$\partial^2 \frac{1}{\Delta x^{2-\varepsilon}} = \frac{4i\pi^{2-\frac{\varepsilon}{2}}}{\Gamma(1-\frac{\varepsilon}{2})}\delta^D(x-x'), \tag{51}$$

one finds

$$\frac{1}{\Delta x^{4-2\varepsilon}} = -\frac{\partial^2}{2\varepsilon(1-\varepsilon)}\Big[\frac{1}{\Delta x^{2-2\varepsilon}} - \frac{\mu^{-\varepsilon}}{\Delta x^{2-\varepsilon}}\Big] - \frac{2\pi^2 i(\sqrt{\pi}\mu)^{-\varepsilon}}{\varepsilon(1-\varepsilon)\Gamma(1-\frac{\varepsilon}{2})}\delta^D(x-x'), \tag{52}$$

where $\mu$ is a parameter that will be used for regularization. Taking $\varepsilon \to 0$ in the first term gives

$$\to -\frac{\partial^2}{4}\Big[\frac{\ln(\mu^2\Delta x^2)}{\Delta x^2}\Big] - \frac{2\pi^2 i(\sqrt{\pi}\mu)^{-\varepsilon}}{\varepsilon(1-\varepsilon)\Gamma(1-\frac{\varepsilon}{2})}\delta^D(x-x')\,. \tag{53}$$

This expression can be used for the first term of (48). Its divergent local part can be canceled by the counterterm Fig.1.(3), which can be rewritten as

$$[^\mu\Pi^\nu]^{(3)}(x,x') = -i\delta Z[^\mu P^\nu](1-\varepsilon\ln(a)+O(\varepsilon^2))\delta^D(x-x')\,, \tag{54}$$

where $\delta Z$ is chosen such that, to the lowest order in $\varepsilon$, the counter-term cancels the divergence in Eq. (53). Note that the logarithm in (54) still gives a contribution to the renormalized vacuum polarization, which grows logarithmically with the scale factor. Thus we find

$$\begin{aligned} i[^\mu\Pi^\nu_{ren}] =& \frac{\alpha}{2\pi^3}\Big\{ -\,[\,^\mu P^\nu]\left[\partial^2\left(\frac{\ln(\mu^2\Delta x^2)}{24\Delta x^2}\right) + \frac{i\pi^2}{3}\ln(a)\delta^4(x-x')\right] \\ & - [\,^\mu P^\nu]\left[\frac{\frac{1}{2}\ln\left(\frac{y}{4}\right) - \frac{1}{2\mathtt{s}} + 2}{\eta\eta'\Delta x^2} + \frac{\ln\left(\frac{y}{4}\right)}{2(y-4)\eta^2\eta'^2}\right] \\ & + [\,^\mu \bar{P}^\nu]\frac{1}{\eta^2\eta'^2}\left[\frac{1}{8}\ln^2\left(\frac{y}{4}\right) + \ln\left(\frac{y}{4}\right)\left(1 - \frac{1}{4\mathtt{s}} + \frac{y-2}{2(y-4)}\right) - \frac{1}{4}Li_2\Big(1-\frac{y}{4}\Big)\right] \\ & + O(\mathtt{s})\Big\}\,, \end{aligned} \tag{55}$$

which is the renormalized vacuum polarization tensor for $D = 4$ written in a manifestly transverse form.

### A. The retarded vacuum polarization tensor

In this section we use the Schwinger-Keldysh formalism [33, 34, 35, 36] to construct the retarded vacuum polarization tensor, $\left[^\mu\Pi^{r,\nu}_{\rm ren}\right]$, required to study the dynamics of photons during inflation. In

order to do that, one is led to modify the Feynman rules of Appendix B, such that, in addition, the vertices become signed as $b = +$ or $-$, while the propagators (22–26) (which connect these vertices) acquire two signed indices, $i\Delta_{bb'}(x,x')\ \ (b,b'=+,-)$. Repeating the procedure of section III, in which we calculated the Feynman vacuum polarization tensor, $i\big[{}^{\mu}\Pi^{\nu}_{++}\big](x,x')$, we shall now outline how to derive other relevant vacuum polarization tensors, $i\big[{}^{\mu}\Pi^{\nu}_{bb'}\big]$. Since the different propagators (22) are the same function of the appropriately modified de Sitter length functions (27), in order to get the different versions of the polarization tensor, we just have to use (27) in (55). A couple of subtle remarks are in order. In the derivation of the vacuum polarization tensor we used $i\Delta(x,x') = i\Delta(x',x)$. While $i\Delta_{++}(x,x')$ and $i\Delta_{--}(x,x')$ are symmetric under the exchange of $x$, $x'$, for the 'off-diagonal' Wightman functions $i\Delta_{+-}$ and $i\Delta_{-+}$, the following symmetry of the propagators ought to be used

$$i\Delta_{bb'}(x,x') = i\Delta_{b'b}(x',x) \qquad (b,b'=+,-)\,, \tag{56}$$

which can be easily established from $\Delta x^2_{bb'}(x,x') = \Delta x^2_{b'b}(x',x)$. Moreover, because the vertices are signed, the off-diagonal vacuum polarization tensors, $i[{}^{\mu}\Pi^{\nu}_{+-}]$ and $i[{}^{\mu}\Pi^{\nu}_{-+}]$, acquire an overall *minus* sign with respect to $i[{}^{\mu}\Pi^{\nu}_{++}]$. Finally, there are no $+-$ or $-+$ seagull graphs or countertems, and there are no local terms coming from Fig. 1.(2) [12]. Based on the above considerations and Eq. (55), we can write,

$$\begin{aligned}
\big[{}^{\mu}\Pi^{\nu}_{bb'}\big] =& \frac{i\alpha}{2\pi^3}bb'\Big\{\big[{}^{\mu}P^{\nu}\big]\bigg[\partial^2\frac{\ln(\mu^2\Delta x^2_{bb'})}{24\Delta x^2_{bb'}} + \frac{i\pi^2}{6}(b+b')\ln(a)\delta^4(x-x')\bigg] \\
&+ \big[{}^{\mu}P^{\nu}\big]\bigg[\frac{\frac{1}{2}\ln(y_{bb'}/4) - \frac{1}{2\mathtt{s}} + 2}{\eta\eta'\Delta x^2_{bb'}} + \frac{\ln(y_{bb'}/4)}{2(y_{bb'}-4)\eta^2\eta'^2}\bigg] \\
&- \big[{}^{\mu}\bar{P}^{\nu}\big]\frac{1}{\eta^2\eta'^2}\bigg[\frac{1}{8}\ln^2\Big(\frac{y_{bb'}}{4}\Big) + \ln\Big(\frac{y_{bb'}}{4}\Big)\Big(1 - \frac{1}{4\mathtt{s}} + \frac{y_{bb'}-2}{2(y_{bb'}-4)}\Big) - \frac{1}{4}Li_2\Big(1-\frac{y_{bb'}}{4}\Big)\bigg]\Big\} \\
&+ O(\mathtt{s})\,.
\end{aligned} \tag{57}$$

For the derivation of an effective field equation of the photon field we shall need the retarded vacuum polarization tensor, which is defined by

$$[{}^{\mu}\Pi^{r;\nu}_{ren}](x,x') = [{}^{\mu}\Pi^{\nu}_{++}](x,x') + [{}^{\mu}\Pi^{\nu}_{+-}](x,x'). \tag{58}$$

Because the two tensors contribute with an overall minus sign (*cf.* the sign prefactor $bb'$ in Eq. (57)), the contributions to the retarded vacuum polarization come only from branch cuts and singularities of (57) in $\Delta x^2_{bb'}$ and $y_{bb'}$ and from the term $\propto (b+b')\ln(a)\delta^4(x-x')$. In order to extract these cut

and pole contributions, the following formulas are useful,

$$\begin{aligned}
\partial^2\left[\frac{\ln(\mu^2\Delta x^2_{bb'})}{24\Delta x^2_{bb'}}\right] &= \partial^4\left[\frac{1}{192}\ln^2(\mu^2\Delta x^2_{bb'}) - \frac{1}{96}\ln(\mu^2\Delta x^2_{bb'})\right] \\
\frac{\frac{1}{2}\ln\left(\frac{y_{bb'}}{4}\right) - \frac{1}{2\mathtt{s}} + 2}{\eta\eta'\Delta x^2_{bb'}} &= \frac{1}{8\eta\eta'}\left[\frac{1}{2}\partial^2(\ln^2(\Delta x^2_{bb'})) + \Big(3 - \ln(4\eta\eta') - \frac{1}{\mathtt{s}}\Big)\partial^2\ln(\Delta x^2_{bb'})\right], \qquad (59)
\end{aligned}$$

from which it can be seen that the contributions, which are singular at the light-cone, yield finite cut and/or pole contributions. The contributions from the logarithms in (57) are simply,

$$\ln(\Delta x^2_{++}) - \ln(\Delta x^2_{+-}) = \ln(\frac{y_{++}}{4}) - \ln(\frac{y_{+-}}{4}) = 2i\pi\,\Theta(\Delta\tau^2)\Theta(\Delta\eta) \qquad (60)$$

$$\ln^2(\frac{y_{++}}{4}) - \ln^2(\frac{y_{+-}}{4}) = 4i\pi\ln\left|\frac{\Delta\tau^2}{4\eta\eta'}\right|\Theta(\Delta\tau^2)\Theta(\Delta\eta) \qquad (61)$$

$$\ln^2(\Delta x^2_{++}) - \ln^2(\Delta x^2_{+-}) = 4i\pi\ln|\Delta\tau^2|\,\Theta(\Delta\tau^2)\Theta(\Delta\eta)\,, \qquad (62)$$

where $\Delta\tau^2 \equiv \Delta\eta^2 - \|\Delta\vec{x}\|^2$, $\Theta$ is the Heaviside step function, $\Theta(\Delta\tau^2) = \Theta(|\Delta\eta| - \|\vec{x}\|)$, $\Delta\eta \equiv \eta - \eta'$ and $\Delta\vec{x} \equiv \vec{x} - \vec{x}'$. From the integral representation of the dilogarithm function one finds,

$$Li_2\Big(1 - \frac{y_{++}}{4}\Big) - Li_2\Big(1 - \frac{y_{+-}}{4}\Big) = -2i\pi\,\ln\Big(1 + \frac{\Delta\tau^2}{4\eta\eta'}\Big)\,\Theta(\Delta\tau^2)\Theta(\Delta\eta). \qquad (63)$$

Combining these equations we find the following expression for the renormalized, retarded vacuum polarization tensor to one-loop order,

$$\begin{aligned}
[^\mu\Pi^{r,\nu}_{ren}](x,x') = \frac{\alpha}{2\pi^2}\Big\{ &-\big[^\mu P^\nu\big]\Big\{\frac{1}{48}\partial^4\Big[\Theta(\Delta\tau^2)\Theta(\Delta\eta)(\ln|\mu^2\Delta\tau^2| - 1)\Big] + \frac{\pi}{3}\ln(a)\delta^4(x-x')\Big\} \\
&- \big[^\mu P^\nu\big]\frac{1}{4\eta\eta'}\Big\{\partial^2\Big[\Theta(\Delta\tau^2)\Theta(\Delta\eta)\ln|\Delta\tau^2|\Big] + \Big(3 - \ln(4\eta\eta') - \frac{1}{\mathtt{s}}\Big)\partial^2\Big[\Theta(\Delta\tau^2)\Theta(\Delta\eta)\Big]\Big\} \\
&- \big[^\mu P^\nu\big]\frac{\Theta(\Delta\tau^2)\Theta(\Delta\eta)}{(\bar{y}-4)\eta^2\eta'^2} \\
&+ \big[^\mu \bar{P}^\nu\big]\frac{\Theta(\Delta\tau^2)\Theta(\Delta\eta)}{\eta^2\eta'^2}\Big\{\frac{1}{2}\Big[\ln\Big|\frac{\bar{y}}{4}\Big| + \ln\Big(1 - \frac{\bar{y}}{4}\Big)\Big] + \Big[\frac{\bar{y}-2}{\bar{y}-4} + 2 - \frac{1}{2\mathtt{s}}\Big]\Big\}\Big\} \\
&+ O(\mathtt{s})\,. \qquad (64)
\end{aligned}$$

## IV. EFFECTIVE FIELD EQUATION AND PHOTON MASS

An effective field equation for photons in de Sitter space-time can be derived by the variation of the Schwinger-Keldysh effective action. The result is the following non-local equation [12],

$$\eta^{\nu\rho}\eta^{\mu\sigma}\partial_\nu F_{\rho\sigma} + \int d^4x'[^\mu\Pi^{r,\nu}_{ren}](x,x')A_\nu(x') + O(A^3) = 0\,. \qquad (65)$$

For simplicity here we shall approximate the retarded vacuum polarization tensor (64) by its leading order $O(\mathtt{s}^{-1})$ contribution,

$$[^\mu\Pi^{r,\nu}_{ren}](x,x') \simeq \frac{\alpha}{8\pi^2\mathtt{s}}\Big[\big[^\mu P^\nu\big]\frac{\partial^2\Theta(\Delta\tau^2)\Theta(\Delta\eta)}{\eta\eta'} - \big[^\mu\bar{P}^\nu\big]\frac{2\Theta(\Delta\tau^2)\Theta(\Delta\eta)}{\eta^2\eta'^2}\Big], \qquad (66)$$

which we shall use to study the effective field equation (65). This is justified provided $|m_\phi^2+\xi R| \ll H^2$. Upon neglecting the $O(A^3)$ contributions in (65), we seek the solutions in the form,

$$A_\nu(x') = \varepsilon_\nu(\vec{k},\eta')e^{i\vec{k}\cdot\vec{x}'}\,, \tag{67}$$

where

$$\Big(\partial_0' - \frac{2}{\eta'}\Big)\varepsilon_0(\vec{k},\eta') = i\vec{k}\cdot\vec{\varepsilon}(\vec{k},\eta')\,, \tag{68}$$

which is equivalent to the generalized Lorentz gauge (12). This gauge is obtained by requiring that divergence of the Proca equation (69) in de Sitter space vanishes, as implied by gauge invariance. Inserting Eqs. (66) and 67) into (65) and evaluating the integral gives the following Proca equation (see Appendix D for the derivation)

$$\eta^{\rho\nu}\partial_\nu(\partial_\rho A_\mu - \partial_\mu A_\rho) - a^2 m_\gamma^2 A_\mu = 0\,, \tag{69}$$

with the photon mass given by

$$\begin{aligned} m_\gamma^2 &= \frac{\alpha H^2}{\pi \mathtt{s}} + O(\mathtt{s}^0) \\ &= \frac{3\alpha H^4}{\pi(m_\phi^2+\xi R)} + O\Big(\Big(\frac{m_\phi^2+\xi R}{H^2}\Big)^0\Big)\,, \end{aligned} \tag{70}$$

where $R$ is the curvature scalar, which in de Sitter space, $R = 12H^2$, and $\xi$ specifies the coupling of the scalar field to gravity. Thus photons that couple to light minimally coupled scalar particles acquire a mass in inflation. A remarkable feature of this result is that, even though $\alpha \ll 1$, such that the one-loop approximation is justified, the photon mass may be much larger than the Hubble parameter.

### A. On the speed of light in inflation

We can use the Proca equation (69) to study the propagation of light in inflation. We seek a solution of the form,

$$A_\mu(x) = \varepsilon_\mu(\vec{k},\eta)e^{i\vec{k}\cdot\vec{x}}\,, \tag{71}$$

where $\varepsilon_\mu(\vec{k},\eta) = (\varepsilon_0(\vec{k},\eta),\vec{\varepsilon}(\vec{k},\eta))$. Thus the $\mu=0$ component of (69) can be written as

$$\varepsilon_0(\vec{k},\eta) = -\frac{i\partial_0\vec{k}\cdot\vec{\varepsilon}(\vec{k},\eta)}{\vec{k}^2 + m_\gamma^2 a^2}, \tag{72}$$

which tells us that the zeroth component of the photon field traces the spatial components. Using the gauge (68) and decomposing $\vec{\varepsilon}$ into the longitudinal and transverse parts

$$\vec{\varepsilon}_L \equiv \frac{(\vec{k}\cdot\vec{\varepsilon})}{\vec{k}^2}\vec{k} \quad , \quad \vec{\varepsilon}_T \equiv \vec{\varepsilon} - \vec{\varepsilon}_L, \tag{73}$$

the spatial components of the Proca equation (69) can be recast as

$$(\partial_0^2 + \vec{k}^2 + m_\gamma^2 a^2)\,\vec{\varepsilon}_T = 0 \tag{74}$$

$$\Big(\partial_0^2 + \vec{k}^2 + m_\gamma^2 a^2 + \frac{2Ha\vec{k}^2}{\vec{k}^2 + m_\gamma^2 a^2}\partial_0\Big)\,\vec{\varepsilon}_L = 0\,. \tag{75}$$

Consider now a longitudinally polarised photon $\vec{A}_L(x) = \vec{\varepsilon}_L e^{i\vec{k}\cdot\vec{x}}$, $\vec{\varepsilon}_L = \vec{\varepsilon}_L(\vec{k},\eta)$. While the magnetic field is trivially equal to zero, $\vec{B}_L = a^{-2}\nabla\times\vec{A}_L = 0$, the electric field does not vanish,

$$\vec{E}_L(x) = -\frac{1}{a^2}\,\frac{1}{1+(k/am_\gamma)^2}\,\partial_\eta\vec{\varepsilon}_L\, e^{i\vec{k}\cdot\vec{x}}\,. \tag{76}$$

From this result one can nicely see that, in the limit when $m_\gamma \to 0$, the electric field vanishes as $\vec{E}_L \propto m_\gamma^2$, rendering the longitudinal polarization unphysical. On the other hand, when $m_\gamma$ grows and becomes comparable to, or larger than, $k/a$, the amplitude of $E_L$ is unsuppressed.

Writing $\vec{\varepsilon}_T$, $\vec{\varepsilon}_L$ as

$$\vec{\varepsilon}_{(T,L)}(\vec{k},\eta) = \vec{c}_{(T,L)}(\vec{k})\,\alpha_{(T,L)}(\vec{k},\eta)\,\exp\Big(-i\int_{\eta_0}^{\eta}\omega_{(T,L)}(\vec{k},\eta')d\eta'\Big), \tag{77}$$

where $\alpha_{(T,L)}$ and $\omega_{(T,L)}$ are real by construction, one obtains from (74) and (75)

$$\frac{\omega_T'}{\omega_T} + 2\frac{\alpha_T'}{\alpha_T} = 0 \quad , \quad \frac{\alpha_T''}{\alpha_T} + \vec{k}^2 + m_\gamma^2 a^2 = \omega_T^2, \tag{78}$$

and

$$\frac{\omega_L'}{\omega_L} + 2\frac{\alpha_L'}{\alpha_L} = -\frac{2Ha\vec{k}^2}{\vec{k}^2 + m_\gamma^2 a^2}, \tag{79}$$

$$\frac{\alpha_L''}{\alpha_L} + \vec{k}^2 + m_\gamma^2 a^2 + \frac{2Ha\vec{k}^2}{\vec{k}^2 + m_\gamma^2 a^2}\frac{\alpha_L'}{\alpha_L} = \omega_L^2, \tag{80}$$

where the *prime* denotes a derivative with respect to $\eta$ ($' \equiv (d/d\eta)$). Solving the first equation of (78) and Eq. (79) gives

$$\alpha_T = \frac{1}{\sqrt{\omega_T}} \quad , \quad \alpha_L = \frac{1}{\sqrt{\omega_L}}\exp\Big(-\int_{\eta_0}^{\eta}\frac{Ha'\vec{k}^2}{\vec{k}^2 + m_\gamma^2 a'^2}d\eta'\Big). \tag{81}$$

Performing the integration in the second equality gives

$$\alpha_L = \frac{1}{\sqrt{\omega_L}}\sqrt{\frac{m_\gamma^2 + (\frac{k}{a})^2}{m_\gamma^2 + (\frac{k}{a_0})^2}}, \tag{82}$$

where $a_0 \equiv a(\eta_0)$. Thus in the oscillatory regime, where one can find real solutions for $\omega_L$, longitudinal modes become suppressed compared to transverse modes during inflation, unless they are non-relativistic, which implies $\|\vec{k}\|/a_0 \ll m_\gamma$. Upon plugging (81) into the second equation of (78) and Eq. (80), we find the following equations for $\omega_T = \omega_T(\vec{k},\eta)$ and $\omega_L = \omega_L(\vec{k},\eta)$

$$\omega_T^2 = \vec{k}^2 + a^2 m_\gamma^2 + \frac{3}{4}\Big(\frac{\omega_T'}{\omega_T}\Big)^2 - \frac{1}{2}\frac{\omega_T''}{\omega_T}\,, \tag{83}$$

$$\omega_L^2 = \vec{k}^2 + a^2 m_\gamma^2 + \frac{3}{4}\Big(\frac{\omega_L'}{\omega_L}\Big)^2 - \frac{1}{2}\frac{\omega_L''}{\omega_L} + \frac{H^2 a^2 \vec{k}^2 (m_\gamma^2 a^2 - 2\vec{k}^2)}{(\vec{k}^2 + m_\gamma^2 a^2)^2}\,. \tag{84}$$

In adiabatic limit, when

$$\omega_{(T,L)}^2 \gg \omega_{(T,L)}'\,, \qquad \omega_{(T,L)}^3 \gg \omega_{(T,L)}''\,, \tag{85}$$

Eqs. (83) and (84) can be iteratively solved. The conditions (85) are satisfied for non-relativistic photons ($k_{\rm ph} \equiv \|\vec{k}\|/a \ll m_\gamma$), when

$$H \ll m_\gamma \qquad\qquad (\texttt{non-relativistic limit}:\ k_{\rm ph} \ll m_\gamma)\,. \tag{86}$$

Then the last term of Eq. (84) can also be neglected. In the relativistic case ($k_{\rm ph} \gg m_\gamma$) we get from (85) ($k_{\rm ph} \gg (Hm_\gamma)^{\frac{1}{2}}$, $(Hm_\gamma^2)^{\frac{1}{3}}$), but this is not sufficient for longitudinal modes. We need the stronger condition

$$\left.\begin{array}{ll} k_{\rm ph} \gg (Hm_\gamma)^{\frac{1}{2}} & (\text{transverse modes}) \\ k_{\rm ph} \gg H & (\text{longitudinal modes}) \end{array}\right\} \quad (\texttt{relativistic limit}:\ k_{\rm ph} \gg m_\gamma)\,, \tag{87}$$

for the adiabatic approximation to work, and the last term in Eq. (84) can be neglected.

When the conditions (86) or (87) are met, one finds from Eqs. (83–84) to leading order in derivatives

$$\omega_{(T,L)} \simeq \omega_0\,, \qquad \omega_0 = \sqrt{\vec{k}^2 + a^2 m_\gamma^2}\,. \tag{88}$$

The propagation speed of massive photons is then given by the group velocity

$$\vec{v}_{\rm group} = \frac{d\omega_{\rm ph}}{d\vec{k}_{\rm ph}} \simeq \frac{\vec{k}_{\rm ph}}{\omega_{\rm ph}}\,, \tag{89}$$

which is always smaller than the speed of light in *vacuo*. When $\|\vec{k}_{\rm ph}\| \ll m_\gamma$, $v_{\rm group}$ can be $\ll 1$. Here we defined the physical frequency, $\omega_{\rm ph} \simeq \omega_0/a$, and the physical wave vector, $\vec{k}_{\rm ph} = \vec{k}/a$.

When adiabatic approximation (85) breaks down, one has to solve for $\omega_{(T,L)} = \omega_{(T,L)}(\vec{k},\eta)$ exactly. As an example, adiabatic approximation for longitudinal relativistic modes breaks down

when $k_{\rm ph}$ approaches $2H$, even if $k_{\rm ph} \gg (Hm_\gamma)^{\frac{1}{2}}$, is satisfied, which means that the derivative terms in Eqs. (83–84) can be neglected. In this case, and when $m_\gamma \ll k_{\rm ph}, H$, Eqs. (84) and (75) take the following form

$$\omega_L^2 \simeq \vec{k}^2 - 2H^2a^2, \tag{90}$$

$$(\partial_0^2 + \vec{k}^2 - 2a^2H^2)a\vec{\varepsilon}_L = 0. \tag{91}$$

From (90) one would not expect oscillatory behavior for $k_{\rm ph} < 2H$, but solving (91) gives

$$\vec{\varepsilon}_L(\vec{k},\eta) = \hat{\vec{k}}\sqrt{k^{-2}+\eta^2}\,\exp\Big\{-i\Big(k\eta + \arctan\big(\frac{1}{k\eta}\big)\Big)\Big\}, \tag{92}$$

which is oscillatory. Here we used $k \equiv \|\vec{k}\|$ and $\hat{\vec{k}} = \vec{k}/k$. Comparing this expression to Eq. (77) one obtains $\omega_L = k - k/(1+\frac{k^2}{H^2a^2})$ for the dispersion relation, such that the group velocity becomes

$$v_{\rm group} = 1 - \frac{1-(k_{\rm ph}/H)^2}{\big[1+(k_{\rm ph}/H)^2\big]^2}\,. \tag{93}$$

This implies that subhorizon photons ($k_{\rm ph} > H$) propagate superluminally, while superhorizon photons ($k_{\rm ph} < H$) are subluminal. When one considers propagation of relativistic photons on subhorizon scales, one finds that longitudinal (transverse) photons propagate superluminally (subluminally). This phenomenon is similar to birefringence [37, 38], where one of the tranverse polarizations may propagate superluminally, and another subluminally.

## V. PHYSICAL IMPLICATIONS OF OUR RESULTS

In this section we study the cosmological consequences of a massive photon in inflation. In the Introduction we argued that a large photon mass during inflation (70) may dramatically influence the photon dynamics, perhaps the most striking being the speed of its propagation: the scalar vacuum fluctuations act as an 'æther', which drags photons, and consequently may dramatically slow down propagation of light.

Another interesting consequence may be creation of magnetic fields on cosmological scales with potentially observable magnitudes [14, 39, 40]. By following the derivation of Ref. [14], we arrive at the following estimate for the (volume-averaged) magnetic field on a (comoving) scale $\ell_c$ [46]:

$$B(t,\ell_c) = \Big(\frac{3\alpha}{\pi z_{\rm eq}}\Big)^{\frac{1}{4}}\Big(\frac{H_0}{2\pi m_\phi c^3}\Big)^{\frac{1}{2}}\frac{H\hbar(1+z)^2}{\ell_c} \qquad \text{(in the Gaussian system)}, \tag{94}$$

$$B(t,\ell_c) = \sqrt{\frac{\mu_0}{4\pi}}\Big(\frac{3\alpha}{\pi z_{\rm eq}}\Big)^{\frac{1}{4}}\Big(\frac{H_0}{2\pi m_\phi c^3}\Big)^{\frac{1}{2}}\frac{H\hbar(1+z)^2}{\ell_c} \qquad \text{(in SI)}\,, \tag{95}$$

where we have reinserted the physical constants. Here $z_{\rm eq} \approx 3200$ denotes the redshift at the matter-radiation equality, $H_0 \simeq 2.3 \times 10^{-18}$ Hz is the present time Hubble parameter (which corresponds to $H_0 \simeq 71\,{\rm km/s/Mpc}$), $H \approx 10^{13}$ GeV/$\hbar$ is the Hubble parameter during inflation, the fine structure constant $\alpha = 1/137$, and $\mu_0$ is the magnetic permeability of vacuum. The result (94) is derived by making the assumption that 1/2 of the energy stored in vacuum fluctuations during inflation is converted into the magnetic energy at the second horizon crossing during radiation era, when also the photon mass is assumed to vanish nonadiabatically. For an alternative derivation we refer to Refs. [39, 40], where continuous matching of the field amplitude and its derivative at the inflation-radiation transition are employed, and the photon is assumed to become massless at the inflation-radiation transition.

Provided the scalar field is sufficiently light, naively it seems that the field strength (94-95) can be significantly larger than the one obtained in Ref. [14], where the photon dynamics coupled to a massless minimally coupled scalar was considered. Taking account of the more recent results obtained in [15] however, which correctly treats the late time asymptotic dynamics of the photon field in inflation, we conclude that one would get equally strong magnetic fields from photons coupled to a minimally coupled massless scalar, provided inflation lasts a sufficiently long time.

Evaluating Eq. (94) with $m_\phi \simeq 100$ GeV [47], we get

$$B(t, \ell_c) \approx 10^{-28} \frac{(1+z)^2}{\ell_c/10kpc} {\rm Gauss}. \tag{96}$$

Assuming galaxy formation at $z \sim 10$ and an amplification by a factor $\sim 80$ through field compression during the collapse of the proto-galaxy [41], one gets field strengths of approximately $B \sim 10^{-24}$ G, for galactic magnetic fields after structure formation at the scales relevant for galactic dynamos. $\ell_c \sim 10$ kpc has been used, which corresponds roughly to $\sim 100$ pc physical length after the field compression at $z \sim 10$. This is most likely sufficient to seed a dynamo mechanism, which generates the micro-Gauss galactic magnetic fields observed today [42, 43].

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## APPENDIX A: EXPANDING THE SCALAR PROPAGATOR

We want to expand the scalar Feynman propagator

$$i\Delta(x,x') = \frac{\Gamma(\frac{D-1}{2}+\nu)\Gamma(\frac{D-1}{2}-\nu)}{(4\pi)^{\frac{D}{2}}\Gamma(\frac{D}{2})} H^{D-2}\, {}_2F_1\Big(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu, \frac{D}{2}, 1-\frac{y}{4}\Big), \tag{A1}$$

in the modified de Sitter length function $y$, $\mathtt{s}$ and $\varepsilon \equiv 4-D$ around $y=0$, $\mathtt{s}=0$ and $\varepsilon=0$, where

$$\nu \equiv \frac{D-1}{2} - \mathtt{s} = \frac{D-1}{2} - \frac{1}{D-1}\frac{m_\phi^2+\xi R}{H^2} + O\left(\Big(\frac{m_\phi^2+\xi R}{H^2}\Big)^2\right). \tag{A2}$$

Using the following transformation formula

$$\begin{aligned} &{}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu, \frac{D}{2}, 1-\frac{y}{4}\right) \\ &= \frac{\Gamma(\frac{D}{2})\Gamma(1-\frac{D}{2})}{\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{1}{2}+\nu)}\, {}_2F_1\left(\frac{D-1}{2}+\nu, \frac{D-1}{2}-\nu, \frac{D}{2}, \frac{y}{4}\right) \\ &+ \frac{1}{(\frac{y}{4})^{\frac{D}{2}-1}} \frac{\Gamma(\frac{D}{2})\Gamma(\frac{D}{2}-1)}{\Gamma(\frac{D-1}{2}-\nu)\Gamma(\frac{D-1}{2}+\nu)}\, {}_2F_1\left(\frac{1}{2}+\nu, \frac{1}{2}-\nu, 2-\frac{D}{2}, \frac{y}{4}\right), \end{aligned} \tag{A3}$$

and the series expansion of the hypergeometric function

$${}_2F_1(a,b,c,z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)}\frac{z^n}{n!}, \tag{A4}$$

one can easily derive

$$\begin{aligned} i\Delta(x,x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \Bigg[ &\frac{\Gamma(1-\frac{D}{2})\Gamma(\frac{D}{2})}{\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{1}{2}+\nu)} \sum_{n=0}^{\infty} \frac{\Gamma(\frac{D-1}{2}+\nu+n)\Gamma(\frac{D-1}{2}-\nu+n)}{\Gamma(\frac{D}{2}+n)} \frac{(\frac{y}{4})^n}{n!} \\ &+ \frac{1}{(\frac{y}{4})^{\frac{D}{2}-1}} \frac{\Gamma(\frac{D}{2}-1)\Gamma(2-\frac{D}{2})}{\Gamma(\frac{1}{2}-\nu)\Gamma(\frac{1}{2}+\nu)} \sum_{n=0}^{\infty} \frac{\Gamma(\frac{1}{2}+\nu+n)\Gamma(\frac{1}{2}-\nu+n)}{\Gamma(2-\frac{D}{2}+n)} \frac{(\frac{y}{4})^n}{n!} \Bigg]. \end{aligned} \tag{A5}$$

Making use of $\Gamma(\frac{1}{2}+b)\Gamma(\frac{1}{2}-b) = \frac{\pi}{\cos(\pi b)}$, $\Gamma(b)\Gamma(1-b) = \frac{\pi}{\sin(\pi b)}$ and inserting in the first equality in (A2) one obtains

$$\begin{aligned} i\Delta(x,x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}} \frac{\cos(\pi(\frac{D-1}{2}-\mathtt{s}))}{\sin(\pi\frac{D}{2})} \Bigg[ &\sum_{n=0}^{\infty} \frac{\Gamma(D-1-\mathtt{s}+n)\Gamma(\mathtt{s}+n)}{\Gamma(\frac{D}{2}+n)} \frac{(\frac{y}{4})^n}{n!} \\ &- \frac{1}{(\frac{y}{4})^{\frac{D}{2}-1}} \sum_{n=0}^{\infty} \frac{\Gamma(\frac{D}{2}-\mathtt{s}+n)\Gamma(1-\frac{D}{2}+\mathtt{s}+n)}{\Gamma(2-\frac{D}{2}+n)} \frac{(\frac{y}{4})^n}{n!} \Bigg]. \end{aligned} \tag{A6}$$

This is still formally exact in $\mathtt{s}$ and $\varepsilon$. Upon pulling the $n=0$ term out of the first sum, the $n=0$ and $n=1$ terms out of the second sum and shifting the second sum we find,

$$\begin{aligned} i\Delta(x,x') = \frac{H^{D-2}}{(4\pi)^{\frac{D}{2}}}\frac{\cos(\pi(\frac{D-1}{2}-\mathtt{s}))}{\sin(\pi\frac{D}{2})}\Bigg[&\sum_{n=1}^{\infty}\frac{\Gamma(D-1-\mathtt{s}+n)\Gamma(\mathtt{s}+n)}{\Gamma(\frac{D}{2}+n)}\frac{(\frac{y}{4})^n}{n!} \\ &-\Big(\frac{y}{4}\Big)^{2-\frac{D}{2}}\sum_{n=1}^{\infty}\frac{\Gamma(\frac{D}{2}-\mathtt{s}+n+1)\Gamma(2-\frac{D}{2}+\mathtt{s}+n)}{\Gamma(3-\frac{D}{2}+n)}\frac{(\frac{y}{4})^n}{(n+1)!} \\ &+\frac{\Gamma(D-1-\mathtt{s})\Gamma(\mathtt{s})}{\Gamma(\frac{D}{2})}-\frac{1}{(\frac{y}{4})^{\frac{D}{2}-1}}\frac{\Gamma(\frac{D}{2}-\mathtt{s})\Gamma(1-\frac{D}{2}+\mathtt{s})}{\Gamma(2-\frac{D}{2})} \\ &-\Big(\frac{y}{4}\Big)^{2-\frac{D}{2}}\frac{\Gamma(\frac{D}{2}-\mathtt{s}+1)\Gamma(2-\frac{D}{2}+\mathtt{s})}{\Gamma(3-\frac{D}{2})}\Bigg]\,. \end{aligned} \tag{A7}$$

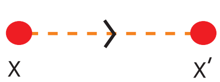
Then expanding in $\mathtt{s}$ gives

$$\begin{aligned} i\Delta(x,x') = \frac{H^{2-\varepsilon}}{(4\pi)^{2-\frac{\varepsilon}{2}}}\Bigg[&\frac{\Gamma(3-\varepsilon)}{\Gamma(2-\frac{\varepsilon}{2})}\Big\{\frac{1}{\mathtt{s}}+\pi\cot\Big(\frac{\pi\varepsilon}{2}\Big)-\gamma_{\rm E}-\psi(3-\varepsilon)+C(\mathtt{s},\varepsilon)\Big\} \\ &-\Big(\frac{y}{4}\Big)^{\frac{\varepsilon}{2}}\frac{\Gamma(3-\frac{\varepsilon}{2})}{\frac{\varepsilon}{2}}\Big\{1+\mathtt{s}\Big(\pi\cot\Big(\frac{\pi\varepsilon}{2}\Big)-\psi\Big(3-\frac{\varepsilon}{2}\Big)+\psi\Big(\frac{\varepsilon}{2}\Big)\Big)\Big\}+\frac{1}{(\frac{y}{4})^{1-\frac{\varepsilon}{2}}}\Gamma\Big(1-\frac{\varepsilon}{2}\Big) \\ &+\sum_{n=1}^{\infty}\Big\{\mathtt{s}\frac{2-n(n+2)\ln(\frac{y}{4})}{n^2}+O(\varepsilon)\Big\}\Big(\frac{y}{4}\Big)^n\Bigg]+O(\mathtt{s}^2)\,, \end{aligned} \tag{A8}$$

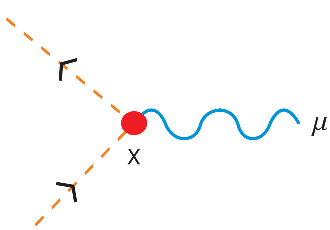
where $\gamma_{\rm E}\approx 0.577$ is the Euler-Mascheroni constant, $\psi$ is defined by $\psi(z)\equiv\Gamma'(z)/\Gamma(z)$ and $C(\mathtt{s},\varepsilon)$ is an order $\mathtt{s}$ term that is independent on the coordinates. The $O(\varepsilon)$ term in the infinite sum, will not be needed for the regularization of the vacuum polarization.

## APPENDIX B: FEYNMAN RULES OF SCALAR QED IN POSITION SPACE

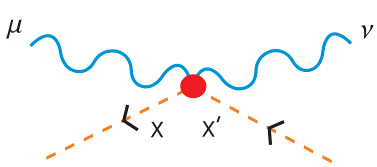
The position space Feynman rules of scalar QED in an arbitrary D-dimensional space with metric $g^{\mu\nu}$ are



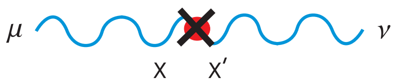

$$= i\Delta(x,x'),$$




$$= e\sqrt{-g(x)}g^{\mu\sigma}(x)(\partial_\sigma^{\rm out}-\partial_\sigma^{\rm in}),$$




$$= -2ie^2\sqrt{-g(x)}g^{\mu\nu}(x)\delta^D(x-x'),$$




$$= -i\delta Z \partial_\rho \big(\sqrt{-g(x)}[g^{\mu\nu}(x) g^{\rho\sigma}(x) - g^{\mu\sigma}(x) g^{\nu\rho}(x)] \partial'_\sigma \delta^D(x - x')\big),$$

where $g \equiv \det[g_{\mu\nu}]$, $\partial_\sigma^{\rm in}$ acts only on the incoming and $\partial_\sigma^{\rm out}$ only on the outgoing propagator. These rules can be derived from the Lagrangean density (1). In de Sitter D-space $i\Delta(x,x')$ and $g_{\mu\nu}(x)$ are given by (36) and (2), respectively, and $\sqrt{-g(x)} = a^D$.

## APPENDIX C: THE 3-POINT AND 4-POINT INTERACTION CONTRIBUTION TO THE ONE-LOOP VACUUM POLARIZATION

The 3-point and 4-point (seagull) interaction contributions to the one-loop vacuum polarization shown in figure 1.(1) and 1.(2) are

$$i[^\mu\Pi^\nu]^{(1)}(x,x') = -2ie^2 a^{D-2} \eta^{\mu\nu} i\Delta(x,x) \delta^D(x - x'), \tag{C1}$$

and

$$i[^\mu\Pi^\nu]^{(2)}(x,x') = 2e^2 a^{D-2} a'^{D-2} \eta^{\mu\rho} \eta^{\nu\sigma} \big[(\partial_\rho i\Delta(x,x'))(\partial'_\sigma i\Delta(x,x')) - i\Delta(x,x') \partial_\rho \partial'_\sigma i\Delta(x,x')\big]. \tag{C2}$$

For $i\Delta(x,x)$ in (C1), Eq. (41) will be used and $i\Delta(x,x')$ is given by (36). The $O(\varepsilon)$ terms, which are not needed for dimensional regularization, can be dropped because they will not contribute when taking D to 4 later. Dropping these and all of $O(\mathtt{s}^2)$ terms, we find for the derivatives of the propagator

$$\partial_\rho i\Delta(x,x') = \Big\{\beta f'(y) + \mathtt{s}\frac{H^2}{(4\pi)^2}[g'(y) + h'(y)]\Big\} \partial_\rho y, \tag{C3}$$

$$\partial'_\sigma i\Delta(x,x') = \Big\{\beta f'(y) + \mathtt{s}\frac{H^2}{(4\pi)^2}[g'(y) + h'(y)]\Big\} \partial'_\sigma y, \tag{C4}$$

$$\begin{aligned}\partial_\rho \partial'_\sigma i\Delta(x,x') =& \Big\{\beta f''(y) + \mathtt{s}\frac{H^2}{(4\pi)^2}[g''(y) + h''(y)]\Big\} (\partial_\rho y)(\partial'_\sigma y) \\ &+ \Big\{\beta f'(y) + \mathtt{s}\frac{H^2}{(4\pi)^2}[g'(y) + h'(y)]\Big\} \partial_\rho \partial'_\sigma y.\end{aligned} \tag{C5}$$

$\beta, f, g$ and $h$ are given in Eqs. (37–40) in section II. Note that the order $1/\mathtt{s}$ of $f(y)$ is constant in $y$. Keeping only terms up to order $\mathtt{s}^0$, we can write the portion of (C2) within the bracket as

$$\begin{aligned}&\big[(\partial_\rho i\Delta(x,x'))(\partial'_\sigma i\Delta(x,x')) - i\Delta(x,x') \partial_\rho \partial'_\sigma i\Delta(x,x')\big] \\ &\quad = \Big[\beta^2(f'^2 - ff'') - 2\frac{H^4}{(4\pi)^4}(g'' + h'')\Big](\partial_\rho y)(\partial'_\sigma y) + \Big[-\beta^2 ff' - 2\frac{H^4}{(4\pi)^4}(g' + h')\Big]\partial_\rho \partial'_\sigma y.\end{aligned} \tag{C6}$$

Differentiating the modified de Sitter length function $y$ gives,

$$\partial_\rho y = aH[y\delta^0_\rho + 2a'H\Delta x_\rho + 2ia'H\,\mathrm{sign}(\eta-\eta')\delta^0_\rho\delta], \tag{C7}$$

$$\partial'_\sigma y = a'H[y\delta^0_\sigma - 2aH\Delta x_\sigma - 2iaH\,\mathrm{sign}(\eta-\eta')\delta^0_\sigma\delta], \tag{C8}$$

$$\begin{aligned}\partial_\rho\partial'_\sigma y =& aa'H^2[y\delta^0_\rho\delta^0_\sigma - 2aH\Delta x_\sigma\delta^0_\rho + 2a'H\Delta x_\rho\delta^0_\sigma - 2\eta_{\rho\sigma} \\ & - 2iaa'H^2|\eta-\eta'|\delta^0_\rho\delta^0_\sigma\delta - 4i\delta(\eta-\eta')\delta^0_\rho\delta^0_\sigma\delta]\,,\end{aligned} \tag{C9}$$

where $\Delta x_\rho \equiv x_\rho - x'_\rho$. When taking $\delta$ to zero only the final order $\delta$ term contributes, and only when it multiplies $f'$

$$-4iaa'H^2\delta(\eta-\eta')\lim_{\delta\to 0} f'(y)\delta = \frac{i}{\beta a^{D-2}}\delta^D(x-x'). \tag{C10}$$

This contribution subtracts off the temporal component of (C1). Dropping $O(\mathtt{s})$ terms and inserting (41) into (C1), we find for the remaining part $i[^\mu\Pi^\nu]^{(1)}$

$$i[^\mu\Pi^\nu_1](x,x') = -2ie^2a^{D-2}\beta\bar{\eta}^{\mu\nu}K\delta^D(x-x'), \tag{C11}$$

where $K$ is the constant part of $f(y)$,

$$K = \frac{2^\varepsilon\Gamma(3-\varepsilon)}{4\Gamma^2(2-\frac{\varepsilon}{2})}\Big[\frac{1}{\mathtt{s}} + \pi\cot\big(\frac{\pi\varepsilon}{2}\big) - \gamma_{\mathrm{E}} - \psi(3-\varepsilon)\Big] = \frac{1}{\varepsilon} + \frac{1}{2\mathtt{s}} - \frac{5}{4} + \ln(2) + O(\varepsilon). \tag{C12}$$

We shall call the left over portion of (C2), $i[^\mu\Pi^\nu_2](x,x')$ and use a bar over the metric tensor to indicate that its zeroth components have been removed, $\bar{\eta}^{\mu\nu} = \eta^{\mu\nu} + \delta^\mu_0\delta^\nu_0$ . Calculating the derivatives of $f(y)$ and dropping some unimportant $O(\varepsilon)$ terms one easily finds

$$f'^2 - ff'' = -\frac{1}{1-\frac{\varepsilon}{2}}\frac{1}{y^{4-\varepsilon}} + \frac{2-\frac{\varepsilon}{2}}{\varepsilon}\frac{1}{y^{3-\varepsilon}} - \frac{(2-\frac{\varepsilon}{2})K}{y^{3-\frac{\varepsilon}{2}}} + \frac{1}{y^2}\Big(\frac{1}{4}\ln\big(\frac{y}{4}\big) - \frac{1}{4\mathtt{s}} + \frac{3}{4}\Big)\,, \tag{C13}$$

and

$$ff' = -\frac{1}{1-\frac{\varepsilon}{2}}\frac{1}{y^{3-\varepsilon}} + \frac{(1-\varepsilon)(2-\frac{\varepsilon}{2})}{2\varepsilon(1-\frac{\varepsilon}{2})}\frac{1}{y^{2-\varepsilon}} - \frac{K}{y^{2-\frac{\varepsilon}{2}}} + \frac{1}{y}\Big(\frac{1}{4}\ln\big(\frac{y}{4}\big) - \frac{1}{4\mathtt{s}} + \frac{1}{2}\Big). \tag{C14}$$

Differentiating (39) and keeping only terms up to order $\varepsilon^0$ yields

$$g'(y) = \frac{3}{y} \qquad , \qquad g''(y) = -\frac{3}{y^2}. \tag{C15}$$

When we perform the summation in (40) and calculate the derivatives of the resulting expression, we obtain

$$h'(y) = \frac{y - 4 + 2(y-6)\ln(\frac{y}{4})}{(y-4)^2}, \tag{C16}$$

$$h''(y) = \frac{48 - 16y + 16y\ln(\frac{y}{4}) + y^2 - 2y^2\ln(\frac{y}{4})}{y(y-4)^3}. \tag{C17}$$

Upon inserting (C7) into (C9) and (C13–C17) into (C6), one gets

$$
\begin{aligned}
i[{}^{\mu}\Pi_2^{\nu}](x,x') =& 2e^2 a^{3-\varepsilon} a'^{3-\varepsilon} H^2 \beta^2 \Bigg[ 4aa'H^2 \Delta x^{\mu} \Delta x^{\nu} \\
& \times \Bigg\{ \frac{1}{1-\frac{\varepsilon}{2}} \frac{1}{y^{4-\varepsilon}} - \frac{2-\frac{\varepsilon}{2}}{\varepsilon} \frac{1}{y^{3-\varepsilon}} + \frac{(2-\frac{\varepsilon}{2})K}{y^{3-\frac{\varepsilon}{2}}} - \frac{1}{y^2} \Big( \frac{1}{4} \ln\big(\frac{y}{4}\big) - \frac{1}{4\mathsf{s}} + \frac{3}{4} \Big) \Bigg\} \\
& - 2\eta^{\mu\nu} \Bigg\{ \frac{1}{1-\frac{\varepsilon}{2}} \frac{1}{y^{3-\varepsilon}} - \frac{(1-\varepsilon)(2-\frac{\varepsilon}{2})}{2\varepsilon(1-\frac{\varepsilon}{2})} \frac{1}{y^{2-\varepsilon}} + \frac{K}{y^{2-\frac{\varepsilon}{2}}} - \frac{1}{y} \Big( \frac{1}{4} \ln\big(\frac{y}{4}\big) - \frac{1}{4\mathsf{s}} + \frac{1}{2} \Big) \Bigg\} \\
& + \Bigg\{ \frac{1}{y^2} \Big( \frac{1}{2} \ln\big(\frac{y}{4}\big) - \frac{1}{2\mathsf{s}} + 2 \Big) + \frac{1}{4y} \Bigg\} \Big( y\delta_0^{\mu}\delta_0^{\nu} - 2a'H\Delta x^{\mu}\delta_0^{\nu} + 2aH\Delta x^{\nu}\delta_0^{\mu} \Big) \Bigg] \\
& - 4e^2 a^3 a'^3 \frac{H^6}{(4\pi)^4} \Bigg[ 4aa'H^2 \Delta x^{\mu}\Delta x^{\nu} \Bigg\{ \frac{3}{y^2} - \frac{48 - 16y + 16y\ln\big(\frac{y}{4}\big) + y^2 - 2y^2 \ln\big(\frac{y}{4}\big)}{y(y-4)^3} \Bigg\} \\
& - 2\eta^{\mu\nu} \Bigg\{ \frac{3}{y} + \frac{y - 4 + 2(y-6)\ln\big(\frac{y}{4}\big)}{(y-4)^2} \Bigg\} \\
& + \frac{2(y-4)(y-8) + 4(12-y)\ln(\frac{y}{4})}{(y-4)^3} \Big( y\delta_0^{\mu}\delta_0^{\nu} - 2a'H\Delta x^{\mu}\delta_0^{\nu} + 2aH\Delta x^{\nu}\delta_0^{\mu} \Big) \Bigg] , \qquad \text{(C18)}
\end{aligned}
$$

where here and in the rest of the appendices upper indices are raised by the Minkowski metric $\eta^{\mu\nu}$. The portion of (C18) within the first bracket and the constant in front of it can be written as

$$
\begin{aligned}
\Rightarrow & - \frac{4e^2\beta^2 H^{2\varepsilon-4}}{1-\frac{\varepsilon}{2}} \Big[ \eta^{\mu\nu} - 2\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{6-2\varepsilon}} \\
& + 2e^2\beta^2 H^{2\varepsilon-2} aa' \frac{2-\frac{\varepsilon}{2}}{\varepsilon(1-\frac{\varepsilon}{2})} \Big[ (1-\varepsilon)\eta^{\mu\nu} - (4-2\varepsilon)\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{4-2\varepsilon}} \\
& - 4e^2\beta^2 H^{\varepsilon-2} a^{1-\frac{\varepsilon}{2}} a'^{1-\frac{\varepsilon}{2}} K \Big[ \eta^{\mu\nu} - (4-\varepsilon)\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{4-\varepsilon}} \\
& + 2e^2\beta^2 a^2 a'^2 \Big[ \big\{ -\frac{1}{Ha}\Delta x^{\mu}\delta_0^{\nu} + \frac{1}{Ha'}\Delta x^{\nu}\delta_0^{\mu} \big\} \big( \ln(\frac{y}{4}) - \frac{1}{\mathsf{s}} + 4 \big) - \Delta x^{\mu}\Delta x^{\nu} \big( \ln(\frac{y}{4}) - \frac{1}{\mathsf{s}} + 3 \big) \Big] \frac{1}{\Delta x^4} \\
& + 2e^2\beta^2 a^2 a'^2 \Big[ \bar{\eta}^{\mu\nu} \big\{ \frac{1}{2}\ln(\frac{y}{4}) - \frac{1}{2\mathsf{s}} + 1 \big\} + \delta_0^{\mu}\delta_0^{\nu} - \frac{1}{2}a'H\Delta x^{\mu}\delta_0^{\nu} + \frac{1}{2}aH\Delta x^{\nu}\delta_0^{\mu} \Big] \frac{1}{\Delta x^2} \\
& + \frac{1}{2} e^2\beta^2 a^3 a'^3 H^2 \delta_0^{\mu}\delta_0^{\nu}. \qquad \text{(C19)}
\end{aligned}
$$

We want to write the vacuum polarization in a form which makes explicit that $\partial_{\mu} i[{}^{\mu}\Pi^{\nu}](x,x') = \partial'_{\nu} i[{}^{\mu}\Pi^{\nu}](x,x') = 0$. To find such a manifestly transverse form we will use the following identities

$$
\Big[ \eta^{\mu\nu} - 2\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{6-2\varepsilon}} = -\frac{1}{2(2-\varepsilon)(3-\varepsilon)} \big[ {}^{\mu}P^{\nu} \big] \frac{1}{\Delta x^{4-2\varepsilon}}, \qquad \text{(C20)}
$$

$$
\Big[ (1-\varepsilon)\eta^{\mu\nu} - (4-2\varepsilon)\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{4-2\varepsilon}} = -\frac{1}{2-2\varepsilon} \big[ {}^{\mu}P^{\nu} \big] \frac{1}{\Delta x^{2-2\varepsilon}}, \qquad \text{(C21)}
$$

$$
\Big[ \eta^{\mu\nu} - (4-\varepsilon)\frac{\Delta x^{\mu}\Delta x^{\nu}}{\Delta x^2} \Big] \frac{1}{\Delta x^{4-\varepsilon}} = -\frac{1}{2-\varepsilon} \big[ {}^{\mu}P^{\nu} \big] \frac{1}{\Delta x^{2-\varepsilon}} - \frac{2i\pi^{2-\frac{\varepsilon}{2}}}{\Gamma(2-\frac{\varepsilon}{2})} \bar{\eta}^{\mu\nu} \delta^D(x-x'). \qquad \text{(C22)}
$$

When inserting (C22) into (C19), the local term exactly cancels $i[^\mu\Pi_1^\nu](x,x')$. We find for the remaining part of (C19),

$$
\begin{aligned}
\Rightarrow \frac{\alpha}{2\pi^3}\Bigg[&\frac{\pi^\varepsilon\Gamma^2(1-\frac{\varepsilon}{2})}{2(3-\varepsilon)}\big[{}^\mu P^\nu\big]\frac{1}{\Delta x^{4-2\varepsilon}} - \frac{1}{\eta\eta'}\big[{}^\mu P^\nu\big]\frac{\frac12\ln\left(\frac{y}{4}\right)-\frac{1}{2\mathtt{s}}+2}{\Delta x^2} \\
&+\frac{1}{\eta^2\eta'^2}\Big\{\big(\eta\Delta x^\mu\delta_0^\nu-\eta'\Delta x^\nu\delta_0^\mu-\Delta x^\mu\Delta x^\nu\big)\Big(\ln\left(\frac{y}{4}\right)-\frac{1}{\mathtt{s}}+3\Big) \\
&\qquad+(\eta-\eta')\big(\eta^{\mu\nu}(\eta-\eta')+\Delta x^\mu\delta_0^\nu+\Delta x^\nu\delta_0^\mu\big)\Big\}\frac{1}{\Delta x^4} \\
&+\frac{1}{\eta^2\eta'^2}\Big\{\bar\eta^{\mu\nu}\Big(\frac12\ln\left(\frac{y}{4}\right)-\frac{1}{2\mathtt{s}}+1\Big)+\delta_0^\mu\delta_0^\nu+\frac{1}{2\eta'}\Delta x^\mu\delta_0^\nu-\frac{1}{2\eta}\Delta x^\nu\delta_0^\mu\Big\}\frac{1}{\Delta x^2}+\frac{\delta_0^\mu\delta_0^\nu}{4\eta^3\eta'^3}\Bigg], \qquad \text{(C23)}
\end{aligned}
$$

where $\alpha = e^2/4\pi$ is the fine structure constant, and we dropped all $O(\varepsilon)$ terms, except in the first line. These terms are not required for regularization. Note that the second term is not yet in manifestly transverse form, because of the factor $\frac{1}{\eta\eta'}$ standing to the left of the projector operator. Bringing it to the right and combining with the rest of (C18) we get for the sum of all terms in Eqs. (C1) and (C2)

$$
\begin{aligned}
i[^\mu\Pi^\nu_{1+2}](x,x') = \frac{\alpha}{2\pi^3}\Bigg[&\frac{\pi^\varepsilon\Gamma^2(1-\frac{\varepsilon}{2})}{2(3-\varepsilon)}\big[{}^\mu P^\nu\big]\frac{1}{\Delta x^{4-2\varepsilon}} - \big[{}^\mu P^\nu\big]\frac{\frac12\ln(\frac{y}{4})-\frac{1}{2\mathtt{s}}+2}{\eta\eta'\Delta x^2} \\
&+\frac{1}{\eta^4\eta'^4y^2}\Big\{(\eta-\eta')\big(\eta^{\mu\nu}(\eta-\eta')+\Delta x^\mu\delta_0^\nu+\Delta x^\nu\delta_0^\mu\big)\Big(\ln\left(\frac{y}{4}\right)-\frac{1}{\mathtt{s}}+4\Big)-\Delta x^\mu\Delta x^\nu\Big(\ln\left(\frac{y}{4}\right)-\frac{1}{\mathtt{s}}+3\Big)\Big\} \\
&+\frac{1}{\eta^3\eta'^3y}\Big\{-2\eta^{\mu\nu}-\delta_0^\mu\delta_0^\nu+\frac{1}{2\eta'}\Delta x^\mu\delta_0^\nu-\frac{1}{2\eta}\Delta x^\nu\delta_0^\mu\Big\}+\frac{\delta_0^\mu\delta_0^\nu}{4\eta^3\eta'^3} \\
&-\frac{\Delta x^\mu\Delta x^\nu}{2\eta^4\eta'^4}\Big\{\frac{3}{y^2}-\frac{48-16y+16y\ln(\frac{y}{4})+y^2-2y^2\ln(\frac{y}{4})}{y(y-4)^3}\Big\} \\
&+\frac{\eta^{\mu\nu}}{4\eta^3\eta'^3}\Big\{\frac{3}{y}+\frac{y-4+2(y-6)\ln(\frac{y}{4})}{(y-4)^2}\Big\} \\
&-\frac{1}{\eta^3\eta'^3}\frac{\frac14(y-4)(y-8)+\frac12(12-y)\ln(\frac{y}{4})}{(y-4)^3}\Big(y\delta_0^\mu\delta_0^\nu+\frac{2}{\eta'}\Delta x^\mu\delta_0^\nu-\frac{2}{\eta}\Delta x^\nu\delta_0^\mu\Big)\Bigg]. \qquad \text{(C24)}
\end{aligned}
$$

We will use the following ansatz for the terms in (C24), which are not in manifestly transverse form

$$
\begin{aligned}
\frac{\alpha}{2\pi^3}\Big[\big[{}^\mu P^\nu\big]\frac{u(y)}{\eta^2\eta'^2}+\big[{}^\mu\bar P^\nu\big]\frac{w(y)}{\eta^2\eta'^2}\Big] = \frac{\alpha}{2\pi^3}\Bigg[&\frac{\delta_0^\mu\delta_0^\nu}{\eta^3\eta'^3}\big\{-4u-5u'y-u''y^2-4w'-4w''y\big\} \\
&+\frac{\eta^{\mu\nu}}{\eta^3\eta'^3}\big\{-4u+u'(6-5y)-u''y^2-4w'+4w''(2-y)\big\} \\
&+\frac{\Delta x^\mu\Delta x^\nu}{\eta^4\eta'^4}\big\{4u''+4w''\big\}+\Big\{\frac{\Delta x^\nu\delta_0^\mu}{\eta^4\eta'^3}-\frac{\Delta x^\mu\delta_0^\nu}{\eta^3\eta'^4}\Big\}4w'' \\
&+\Big\{\frac{\Delta x^\mu\delta_0^\nu}{\eta^4\eta'^3}-\frac{\Delta x^\nu\delta_0^\mu}{\eta^3\eta'^4}-\frac{\eta^{\mu\nu}}{\eta^4\eta'^2}-\frac{\eta^{\mu\nu}}{\eta^2\eta'^4}\Big\}\big\{6u'+2u''y+4w''\big\}\Bigg]. \qquad \text{(C25)}
\end{aligned}
$$

Comparing this to the relevant terms in (C24), one finds

$$4w'' = -\frac{1}{y^2}\Big(\ln\big(\frac{y}{4}\big) - \frac{1}{\mathsf{s}} + 4\Big) - \frac{1}{2y} + 2\frac{\frac{1}{4}(y-4)(y-8) + \frac{1}{2}(12-y)\ln(\frac{y}{4})}{(y-4)^3}, \tag{C26}$$

$$6u' + 2u''y + 4w'' = -\frac{1}{y^2}\Big(\ln\big(\frac{y}{4}\big) - \frac{1}{\mathsf{s}} + 4\Big), \tag{C27}$$

$$4u'' + 4w'' = -\frac{1}{y^2}\Big(\ln\big(\frac{y}{4}\big) - \frac{1}{\mathsf{s}} + 3\Big) - \frac{1}{2}\Big\{\frac{3}{y^2} - \frac{48 - 16y + 16y\ln(\frac{y}{4}) + y^2 - 2y^2\ln(\frac{y}{4})}{y(y-4)^3}\Big\}, \tag{C28}$$

$$\begin{aligned} 4u - u'(6-5y) + u''y^2 + 4w' - 4w''(2-y) = & \frac{2}{y^2}\big(\ln(\frac{y}{4}) - \frac{1}{\mathsf{s}} + 4\big) + \frac{2}{y} \\ & - \frac{1}{4}\Big\{\frac{3}{y} + \frac{y - 4 + 2(y-6)\ln(\frac{y}{4})}{(y-4)^2}\Big\}, \end{aligned} \tag{C29}$$

$$4u + 5u'y + u''y^2 + 4w' + 4w''y = \frac{1}{y} - \frac{1}{4} + y\frac{\frac{1}{4}(y-4)(y-8) + \frac{1}{2}(12-y)\ln(\frac{y}{4})}{(y-4)^3}. \tag{C30}$$

One can find the following solutions to these differential equations

$$u(y) = -\frac{\ln(\frac{y}{4})}{2(y-4)}, \tag{C31}$$

$$w(y) = \frac{1}{8}\ln^2\big(\frac{y}{4}\big) + \ln\big(\frac{y}{4}\big)\Big(1 - \frac{1}{4\mathsf{s}} + \frac{y-2}{2(y-4)}\Big) - \frac{1}{4}Li_2\big(1 - \frac{y}{4}\big), \tag{C32}$$

where the dilogarithm function, defined by $Li_2(z) \equiv -\int_0^z \frac{\ln(1-t)}{t}dt$ appears. Upon inserting these solutions into (C25) and combining the result with the first two terms of (C24), one obtains the following expression for the graphs in Fig.1.(1) and Fig.1.(2)

$$\begin{aligned} i[^\mu\Pi^\nu_{1+2}](x,x') = \frac{\alpha}{2\pi^3}\Bigg[ & [^\mu P^\nu]\Big\{\frac{\pi^\varepsilon\Gamma^2(1-\frac{\varepsilon}{2})}{2(3-\varepsilon)}\frac{1}{\Delta x^{4-2\varepsilon}} - \frac{\frac{1}{2}(\ln(\frac{y}{4}) - \frac{1}{\mathsf{s}} + 2) + 1}{\eta\eta'\Delta x^2} - \frac{\ln\big(\frac{y}{4}\big)}{2(y-4)\eta^2\eta'^2}\Big\} \\ & + [^\mu \bar{P}^\nu]\frac{\frac{1}{8}\ln^2\big(\frac{y}{4}\big) + \ln\big(\frac{y}{4}\big)\big(1 - \frac{1}{4\mathsf{s}} + \frac{y-2}{2(y-4)}\big) - \frac{1}{4}Li_2\big(1 - \frac{y}{4}\big)}{\eta^2\eta'^2} + O(\mathsf{s})\Bigg]. \end{aligned} \tag{C33}$$

## APPENDIX D: INTEGRATION OF THE RETARDED VACUUM POLARIZATION AGAINST THE PHOTON FIELD

In order to simplify the effective field equation for the photon field, we need to evaluate the integral

$$\int d^4x'[^\mu\Pi^{r;\nu}_{ren}](x,x')A_\nu(x'), \tag{D1}$$

where

$$[^{\mu}\Pi^{r,\nu}_{ren}](x,x') = \frac{\alpha}{8\pi^2 \mathtt{s}}\Big[\big[{}^{\mu}P^{\nu}\big]\frac{\partial^2\Theta(|\Delta\eta| - \|\Delta\vec{x}\|)\Theta(\Delta\eta)}{\eta\eta'} - \big[{}^{\mu}\bar{P}^{\nu}\big]\frac{2\Theta(|\Delta\eta| - \|\Delta\vec{x}\|)\Theta(\Delta\eta)}{\eta^2\eta'^2}\Big]. \quad \text{(D2)}$$

and

$$A_\nu(x') = \varepsilon_\nu(\vec{k},\eta')e^{i\vec{k}\cdot\vec{x}'}, \quad \text{(D3)}$$

$$\Big(\partial'_0 - \frac{2}{\eta'}\Big)\varepsilon_0(\vec{k},\eta') = i\vec{k}\cdot\vec{\varepsilon}(\vec{k},\eta')\,. \quad \text{(D4)}$$

We can simplify the integral by using the identity,

$$\begin{aligned}\big[{}^{\mu}P^{\nu}\big]\frac{f(x-x')}{\eta\eta'} &= \Big\{\bar{\eta}^{\mu\nu}\big[-\frac{1}{\eta\eta'}\partial^2 + \frac{\Delta\eta}{\eta^2\eta'^2}\partial_0 - \frac{1}{\eta^2\eta'^2}\big] + \frac{\delta^\mu_0\delta^\nu_0}{\eta\eta'}\vec{\nabla}^2 \\ &\quad - \delta^\nu_0\bar{\partial}^\mu\big[\frac{1}{\eta\eta'}\partial_0 - \frac{1}{\eta^2\eta'}\big] + \big[\frac{1}{\eta\eta'}\bar{\partial}^\mu - \frac{\delta^\mu_0}{\eta\eta'}\partial_0 - \frac{\delta^\mu_0}{\eta\eta'^2}\big]\bar{\partial}^\nu\Big\}f(x-x'). \end{aligned} \quad \text{(D5)}$$

where $f$ can be an arbitrary function of $x - x'$ and a bar over a tensor is used to indicate that its zero components have been removed, *e.g.* $\bar{\eta}^{\mu\nu} \equiv \eta^{\mu\nu} + \delta^\mu_0\delta^\nu_0$, $\bar{\partial}^\mu \equiv \partial^\mu - \delta^\mu_0\partial^0$. Using this we find for the integral (D1)

$$\begin{aligned}\Rightarrow \frac{\alpha}{8\pi^2\mathtt{s}}\int d^4x'\Bigg[&\Big\{\bar{\eta}^{\mu\nu}\big[-\frac{1}{\eta\eta'}\partial^2 + \frac{\Delta\eta}{\eta^2\eta'^2}\partial_0 - \frac{1}{\eta^2\eta'^2}\big] + \frac{\delta^\mu_0\delta^\nu_0}{\eta\eta'}\vec{\nabla}^2 - \delta^\nu_0\bar{\partial}^\mu\big[\frac{1}{\eta\eta'}\partial_0 - \frac{1}{\eta^2\eta'}\big] \\ &+ \big[\frac{1}{\eta\eta'}\bar{\partial}^\mu - \frac{\delta^\mu_0}{\eta\eta'}\partial_0 - \frac{\delta^\mu_0}{\eta\eta'^2}\big]\bar{\partial}^\nu\Big\}\partial^2\Theta(|\Delta\eta| - \|\Delta\vec{x}\|)\Theta(\Delta\eta) \\ &-\frac{2}{\eta^2\eta'^2}\big[{}^{\mu}\bar{P}^{\nu}\big]\Theta(|\Delta\eta| - \|\Delta\vec{x}\|)\Theta(\Delta\eta)\Bigg]\varepsilon_\nu(\vec{k},\eta')e^{i\vec{k}\cdot\vec{x}'}. \end{aligned} \quad \text{(D6)}$$

In the last term the transverse projector $\big[{}^{\mu}\bar{P}^{\nu}\big]$ hits a function of $x - x'$ only. Therefore it can be replaced by $-(\bar{\eta}^{\mu\nu}\vec{\nabla}^2 - \bar{\partial}^\mu\bar{\partial}^\nu)$. Then this expression can be recast as

$$\begin{aligned}\frac{\alpha}{8\pi^2\mathtt{s}}\Bigg[&-\frac{\bar{\eta}^{\mu\nu}}{\eta}\partial^4 I_{1,\nu}(x) + \frac{\bar{\eta}^{\mu\nu}}{\eta}\partial_0\partial^2 I_{2,\nu}(x) - \frac{\bar{\eta}^{\mu\nu}}{\eta^2}\partial_0\partial^2 I_{1,\nu}(x) - \frac{\bar{\eta}^{\mu\nu}}{\eta^2}\partial^2 I_{2,\nu}(x) \\ &+ \frac{\delta^\mu_0\delta^\nu_0}{\eta}\vec{\nabla}^2\partial^2 I_{1,\nu}(x) - \frac{\delta^\nu_0}{\eta}\bar{\partial}^\mu\partial_0\partial^2 I_{1,\nu}(x) + \frac{\delta^\nu_0}{\eta^2}\bar{\partial}^\mu\partial^2 I_{1,\nu}(x) \\ &+ \frac{1}{\eta}\bar{\partial}^\mu\bar{\partial}^\nu\partial^2 I_{1,\nu}(x) - \frac{\delta^\mu_0}{\eta}\partial_0\bar{\partial}^\nu\partial^2 I_{1,\nu}(x) - \frac{\delta^\mu_0}{\eta}\bar{\partial}^\nu\partial^2 I_{2,\nu}(x) \\ &+ \frac{2\bar{\eta}^{\mu\nu}}{\eta^2}\vec{\nabla}^2 I_{2,\nu}(x) - \frac{2}{\eta^2}\bar{\partial}^\mu\bar{\partial}^\nu I_{2,\nu}(x)\Bigg], \end{aligned} \quad \text{(D7)}$$

where

$$I_{(1,2),\nu}(x) = \int d^4x'\frac{1}{\eta'^{(1,2)}}\Theta(|\Delta\eta| - \|\Delta\vec{x}\|)\Theta(\Delta\eta)\varepsilon_\nu(\vec{k},\eta')e^{i\vec{k}\cdot\vec{x}'}. \quad \text{(D8)}$$

One can get rid of the step functions by choosing appropriate limits of the integration. Then one can rewrite (D8) as

$$= e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{4\pi\varepsilon_\nu(\vec{k},\eta')}{\eta'^{(1,2)}} \int_0^{\Delta\eta} r^2 dr \int_{-1}^{+1} \frac{d\cos\theta}{2}\, e^{-ikr\cos\theta}, \tag{D9}$$

where $\theta$ is the angle between $\Delta\vec{x}$ and $\vec{k}$, $r = \|\Delta\vec{x}\|$ and $k = \|\vec{k}\|$. Performing the $r$ and $\cos\theta$ integration yields

$$= e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^{(1,2)}} \frac{4\pi}{k^3}\big(\sin(k\Delta\eta) - (k\Delta\eta)\cos(k\Delta\eta)\big). \tag{D10}$$

From this expression we obtain

$$\partial^2 I_{(1,2),\nu}(x) = -e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^{(1,2)}} \frac{8\pi}{k}\sin(k\Delta\eta). \tag{D11}$$

Using (D11) one can write the contributions to the $\mu = 0$ component of Eq. (D7) as

$$\begin{aligned}\Rightarrow \frac{\alpha}{\pi\mathsf{s}}\frac{\delta_0^\mu}{\eta}\Bigg[&k^2 e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{\varepsilon_0(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta)\\ &+i\bar{k}^\nu \partial_0 e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{\bar{\varepsilon}_\nu(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta)\\ &+i\bar{k}^\nu e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \frac{\bar{\varepsilon}_\nu(\vec{k},\eta')}{\eta'^2}\frac{1}{k}\sin(k\Delta\eta)\Bigg].\end{aligned} \tag{D12}$$

Upon using (D4) and performing partial integration this can be recast as

$$\begin{aligned}\Rightarrow \frac{\alpha}{\pi\mathsf{s}}\frac{\delta_0^\mu}{\eta}\Bigg[&k^2 e^{i\vec{k}\cdot\vec{x}} \int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\frac{\sin(k\Delta\eta)}{\eta'}\\ &+e^{i\vec{k}\cdot\vec{x}}\partial_0\Big\{\int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\Big(\frac{k\cos(k\Delta\eta)}{\eta'} + \frac{\sin(k\Delta\eta)}{\eta'^2}\Big) - 2\int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\frac{\sin(k\Delta\eta)}{\eta'^2}\Big\}\\ &+e^{i\vec{k}\cdot\vec{x}}\Big\{\int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\Big(\frac{k\cos(k\Delta\eta)}{\eta'^2} + \frac{2\sin(k\Delta\eta)}{\eta'^3}\Big) - 2\int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\frac{\sin(k\Delta\eta)}{\eta'^3}\Big\}\Bigg].\end{aligned}$$

When performing the $\partial_0$ differentiation all integrals cancel, only one term remains. It comes from the differentiation with respect to the upper limit of the integral $\int_{-\infty}^{\eta} d\eta' \varepsilon_0(\vec{k},\eta')\frac{1}{k}\frac{k\cos(k\Delta\eta)}{\eta'}$, thus we get

$$\Rightarrow -\frac{\alpha}{\pi\mathsf{s}}\frac{\delta_0^\mu}{\eta^2}\varepsilon^0(\vec{k},\eta)e^{i\vec{k}\cdot\vec{x}}\,. \tag{D13}$$

The terms contributing to the spatial components of (D7) can be written as

$$
\begin{aligned}
\frac{\alpha}{\pi \mathtt{s}}\Bigg[ &-\frac{\bar{\eta}^{\mu\nu}}{\eta}(\partial_0^2+k^2)e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta) \\
&-\frac{\bar{\eta}^{\mu\nu}}{\eta}\partial_0 e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^2}\frac{1}{k}\sin(k\Delta\eta) \\
&+\frac{\bar{\eta}^{\mu\nu}}{\eta^2}\partial_0 e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta) \\
&+\frac{\bar{\eta}^{\mu\nu}}{\eta^2}e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^2}\frac{1}{k}\sin(k\Delta\eta) \\
&+\frac{1}{\eta}(i\bar{k}^\mu)\partial_0 e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_0(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta) \\
&-\frac{1}{\eta^2}(i\bar{k}^\mu)e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_0(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta) \\
&-\frac{1}{\eta}(i\bar{k}^\mu)(i\bar{k}^\nu)e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'}\frac{1}{k}\sin(k\Delta\eta) \\
&-\frac{\bar{\eta}^{\mu\nu}}{\eta^2}e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^2}\frac{1}{k}\big(\sin(k\Delta\eta)-(k\Delta\eta)\cos(k\Delta\eta)\big) \\
&-\frac{1}{\eta^2}(i\bar{k}^\mu)(i\bar{k}^\nu)e^{i\vec{k}\cdot\vec{x}}\int_{-\infty}^{\eta}d\eta'\frac{\varepsilon_\nu(\vec{k},\eta')}{\eta'^2}\frac{1}{k^3}\big(\sin(k\Delta\eta)-(k\Delta\eta)\cos(k\Delta\eta)\big)\Bigg],
\end{aligned}
\tag{D14}
$$

where (D10) and (D11) have been used. We now insert Eq. (D4) into the terms in which $i\bar{k}^\nu\varepsilon_\nu(\vec{k},\eta')$ appears and partially integrate. When we then perform the remaining differentiations all integrals cancel, and only a term, which comes from the differentiation with respect to the upper limit of the first integral, remains. This term equals

$$\Rightarrow -\frac{\alpha}{\pi \mathtt{s}}\frac{1}{\eta^2}\bar{\varepsilon}^\mu(\vec{k},\eta)e^{i\vec{k}\cdot\vec{x}}. \tag{D15}$$

Combining this with (D13) we finally find for the integral (D1)

$$\int d^4x'[{}^\mu\Pi^{r,\nu}_{ren}](x,x')A_\nu(x') = -\frac{\alpha H^2}{\pi \mathtt{s}}a^2\eta^{\mu\nu}A_\nu(x). \tag{D16}$$

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[46] This estimate is by a factor $(8\pi)^{1/2}$ times larger than the result in [14]. The difference can be traced to the factor $(8\pi)^{-1}$ in the definition of the electromagnetic energy density in the Gaussian system of units.

[47] The scalar mass of electroweak scale implies a very large (super-Planckian) photon mass, $m_\gamma \sim 10^{22}$ GeV. A more realistic estimate for the photon mass in this case would be $m_\gamma \sim M_{\rm Pl} \sim 10^{19}$ GeV, which formally corresponds to $m_\phi \sim 300$ TeV. When the Planck mass is used in (94), one gets about two orders of magnitude weaker seed field.